Monmouth COLLEGE

- Name: ________________
- Date: ________________
- Section: ________________

ECON 300: Intermediate Price Theory

Problem Set #4 - Part #2: Suggested Solutions

Fall 2024

Problem 1. Deriving the Engel Curve

Suppose that the consumer is participating in a market consisting of goods x and y. The price of good x is 2, the price of good y is 1. The consumer's utility function is given as follows:

$$u(x, y) = x + 2y$$

1.A. If the consumer's income is $M_0 = 2$, what is the consumer's optimal consumption bundle?

We are dealing with perfect substitutes (linear utility function), so we apply the "per-dollar marginal utility" approach to find that the consumer should exclusively purchase good y.

$$\frac{MU_x}{P_x} = \frac{1}{2} < \frac{2}{1} = \frac{MU_y}{P_y}$$

So, the consumer should purchase 2 units of good y.[1]

$$\boxed{y^* = 2}$$

1.B. If the consumer's income is $M_1 = 6$, what is the consumer's optimal consumption bundle?

Since the consumer's preferences have not changed (utility function remains unchanged), and prices have not changed, the consumer still exclusively purchases good y. So, the consumer should purchase 6 units of good y.

$$\boxed{y^* = 6}$$

1.C. If the consumer's income is $M_2 = 10$, what is the consumer's optimal consumption bundle?

Since the consumer's preferences have not changed (utility function remains unchanged), and prices have not changed, the consumer still exclusively purchases good y. So, the consumer should purchase 10 units of good y.

$$\boxed{y^* = 10}$$

1.D. If the consumer's income is $M_3 = 14$, what is the consumer's optimal consumption bundle?

Since the consumer's preferences have not changed (utility function remains unchanged), and prices have not changed, the consumer still exclusively purchases good y. So, the consumer should purchase 14 units of good y.

$$\boxed{y^* = 14}$$

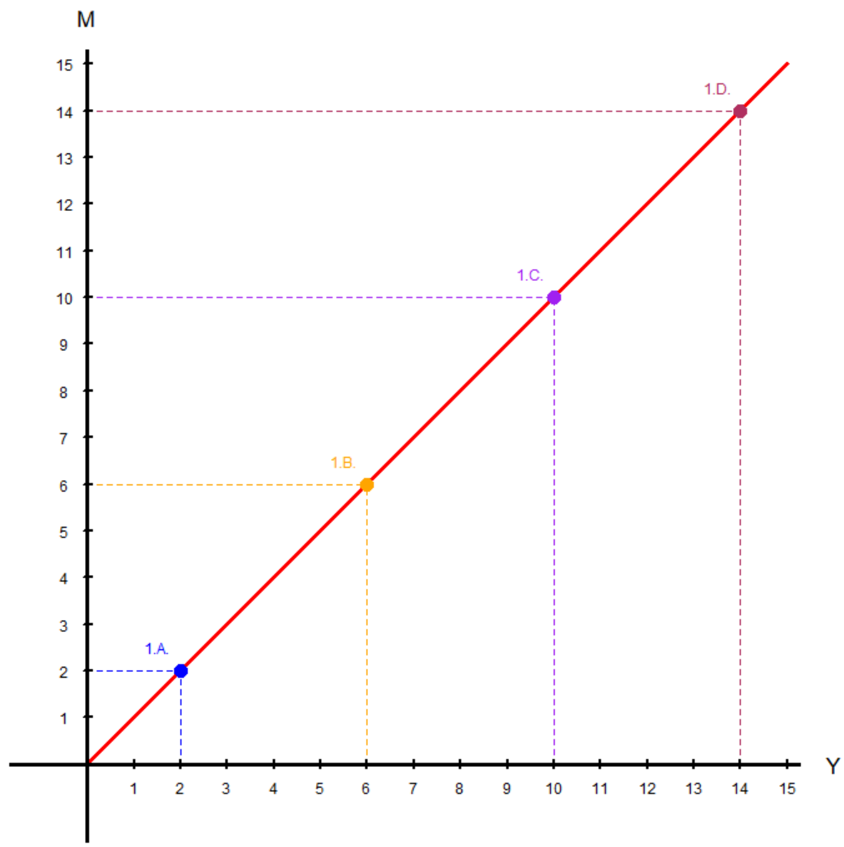
[1] $\frac{M_0}{P_y} = 2$

Problem 1. Deriving the Engel Curve (continued)

Suppose that the consumer is participating in a market consisting of goods x and y. The price of good x is 2, the price of good y is 1. The consumer's utility function is given as follows:

$$u(x, y) = x + 2y$$

1.E. Using your answers from `1.A` through `1.D`, plot the consumer's Engel curve for good y in the diagram below.



1.F. Is good y a Normal good or an Inferior good? Why?

Since the consumer is increasing their consumption of good y as their income increases, good y is a normal good.

Problem 2. Individual and Market Demand

Suppose we have a market for good x with two consumers. Their individual demand functions are:

- Consumer 1: $x_1 = 10 - \frac{1}{4}P_x$
- Consumer 2: $x_2 = 20 - \frac{2}{3}P_x$

2.A Find the inverse demand function for consumer 1.

Rearrange the formula so that it leaves only P_x on the left hand side.

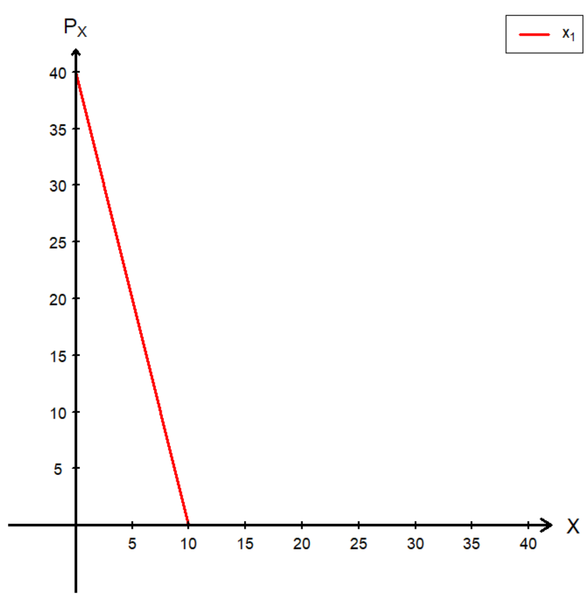
$$x_1 = 10 - \frac{1}{4}P_x \quad \Rightarrow \quad \frac{1}{4}P_x = 10 - x_1 \quad \Rightarrow \quad \boxed{P_x = 40 - 4x_1}$$

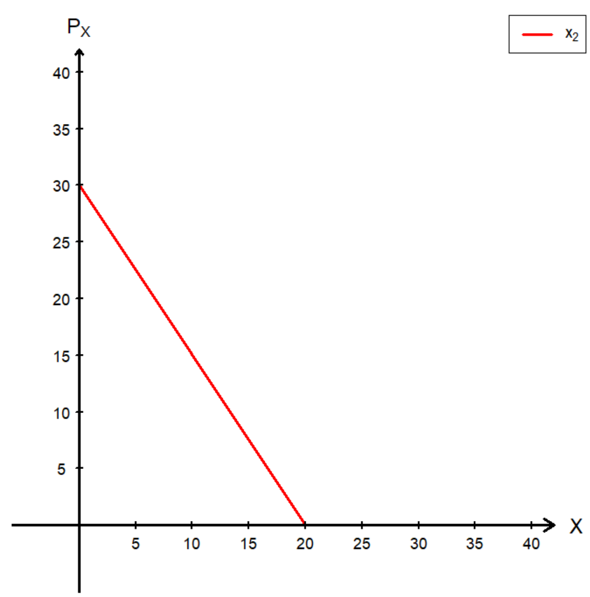
2.B Find the inverse demand function for consumer 2.

Rearrange the formula so that it leaves only P_x on the left hand side.

$$x_2 = 10 - \frac{2}{3}P_x \quad \Rightarrow \quad \frac{2}{3}P_x = 20 - x_2 \quad \Rightarrow \quad \boxed{P_x = 30 - \frac{3}{2}x_2}$$

2.C Plot the consumer 1's demand curve to the left, and consumer 2's demand curve to the right.



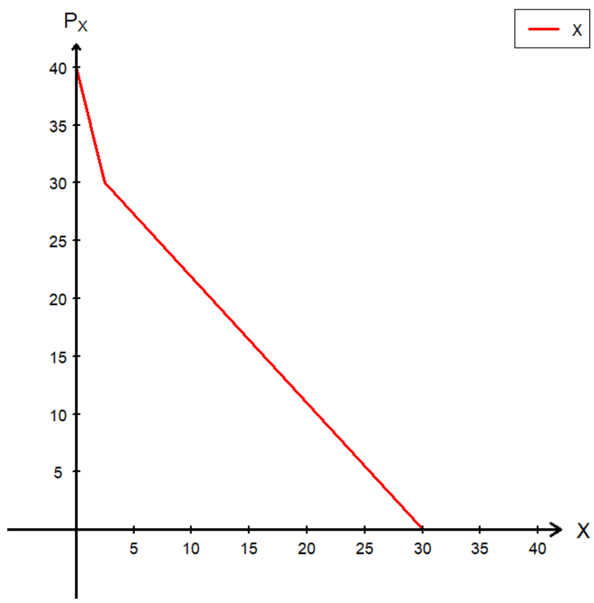


Problem 2. Individual and Market Demand (continued)

Suppose we have a market for good x with two consumers. Their individual demand functions are:

- Consumer 1: $x_1 = 10 - \frac{1}{4}P_x$
- Consumer 2: $x_2 = 20 - \frac{2}{3}P_x$

2.D. Plot the market demand for good x.



2.E. (ADVANCED) Find the formula for the market demand.

The market demand will depend on the market prices. Based on the results from 3.D, we can see that until consumer 2 begins to purchase good x, the market consists of only consumer 1. So, for prices above 30, the market demand is the individual demand of consumer 1. Meanwhile, when the market price dips below 30, then consumer 2 begins to purchase good x, so the market demand for market prices below 30 should be the (horizontal) sum of the two consumers. So, the inverse market demand should be:

$$X(P_x; P_{-x}, M) = \begin{cases} 10 - \frac{1}{4}P_x, & \text{if } 30 < P_x \leq 40 \\ \frac{140}{11} - \frac{2}{11}P_x, & \text{if } 0 \leq P_x \leq 30 \end{cases}$$

- Score: ________________
- Extra Credit: ________________