Monmouth COLLEGE

- Name: ____________________
- Date: ____________________
- Section: ____________________

ECON 300: Intermediate Price Theory

Problem Set #4 - Part #1: Suggested Solutions

Fall 2024

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Problem 1. Comparative Statics of the UMP: Price

Suppose that a consumer's utility function $u(\cdot)$ over two goods x and y is given as:

$$u(x, y) = 2xy^4$$

The consumer's budget is \$120, and the unit price of good x is \$1, and the unit price of good y is \$4.

1.A. Find the marginal utility of good x and y.

Take the partial derivative of the utility function with respect to x and y.

$$MU_x = \frac{\partial}{\partial x}u(x, y) = \frac{\partial}{\partial x}2xy^4 = \boxed{2y^4} \qquad MU_y = \frac{\partial}{\partial y}u(x, y) = \frac{\partial}{\partial y}2xy^4 = \boxed{8xy^3}$$

1.B. Find the marginal rate of substitution between goods x and y.

The marginal rate of substitution is the ratio of the marginal utilities of good x and y.

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{2y^4}{8xy^3} = \boxed{\frac{y}{4x}}$$

1.C. Find the formal expression for the consumer's budget constraint.

The budget constraint is expressed as follows:

$$\texttt{Money Spent on Goods} = \texttt{Total Budget} \quad \Rightarrow \quad \boxed{x + 4y = 120}$$

1.D. Find the optimal ratio of goods x and y the consumer should purchase to maximize their utility.

The optimal ratio of goods x and y can be found using the relative prices of goods (slope of the budget line) and the marginal rate of substitution (slope of the indifference curve).

$$MRS_{xy} = \frac{P_x}{P_y} \quad \Rightarrow \quad \frac{y}{4x} = \frac{1}{4} \quad \Rightarrow \quad 4y = 4x \quad \Rightarrow \quad \boxed{x = y}$$

1.E. Find the optimal bundle that the consumer should purchase to maximize their utility.

Use the budget constraint from `1.C.`, and substitute in the optimal ratio from `1.D.` to find the optimal bundle.

$$\underbrace{x + 4y = 120 \quad \Rightarrow \quad x + 4x = 120}_{\because x=y \text{ from 1.D}} \quad \Rightarrow \quad 5x = 120 \quad \Rightarrow \quad \underbrace{\boxed{x^* = 24} \quad \Rightarrow \quad \boxed{y^* = 24}}_{\because x=y \text{ from 1.D}}$$

Problem 1. Comparative Statics of the UMP: Price (continued)

1.F. Suppose that the price of good x increased from \$1 to \$2. Find the optimal bundle that the consumer should purchase to maximize their utility under this updated price of good x.

Consumer's MRS is unchanged, and under the new price of good x, P_x^1, the optimal ratio is:

$$MRS_{xy} = \frac{P_x^1}{P_y} \quad \Rightarrow \quad \frac{y}{4x} = \frac{2}{4} \quad \Rightarrow \quad 4y = 8x \quad \Rightarrow \quad \boxed{2x = y}$$

Using the updated budget constraint, and the new optimal ratio, we find:

$$2x + 4y = 120 \quad \Rightarrow \quad y + 4y = 120 \quad \Rightarrow \quad 5y = 120 \quad \Rightarrow \quad \boxed{y^* = 24} \quad \Rightarrow \quad \boxed{x^* = 12}$$

1.G. Suppose that the price of good x increased from \$2 to \$4. Find the optimal bundle that the consumer should purchase to maximize their utility under this updated price of good x.

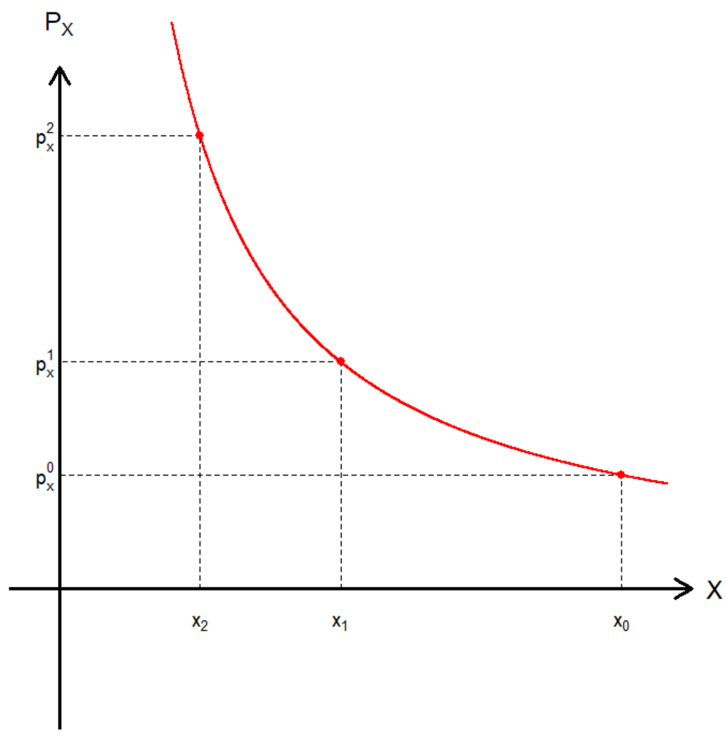
Following the same process as `1.F` with new price of x, P_x^2:

$$MRS_{xy} = \frac{P_x^2}{P_y} \quad \Rightarrow \quad \frac{y}{4x} = \frac{4}{4} \quad \Rightarrow \quad \boxed{4x = y}$$

Using the updated budget constraint, and the new optimal ratio, we find:

$$4x + 4y = 120 \quad \Rightarrow \quad y + 4y = 120 \quad \Rightarrow \quad 5y = 120 \quad \Rightarrow \quad \boxed{y^* = 24} \quad \Rightarrow \quad \boxed{x^* = 6}$$

1.H. Using your answers from `1.E`, `1.F`, and `1.G`, approximate the consumer's demand curve in the empty chart below.



Problem 2. Comparative Statics of the UMP: Income

Suppose that a consumer's utility function $u(\cdot)$ over two goods x and y is given as:

$$u(x, y) = 5x^2y$$

The consumer's budget is \$60, and the unit price of good x is \$1, and the unit price of good y is \$2.

2.A. Find the marginal utility of good x and y.

Take the partial derivative of the utility function with respect to x and y.

$$MU_x = \frac{\partial}{\partial x}u(x, y) = \frac{\partial}{\partial x}5x^2y = \boxed{10xy} \qquad MU_y = \frac{\partial}{\partial y}u(x, y) = \frac{\partial}{\partial y}5x^2y = \boxed{5x^2}$$

2.B. Find the marginal rate of substitution between goods x and y.

The marginal rate of substitution is the ratio of the marginal utilities of good x and y.

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{10xy}{5x^2} = \boxed{\frac{2y}{x}}$$

2.C. Find the formal expression for the consumer's budget constraint.

The budget constraint is expressed as follows:

$$\texttt{Money Spent on Goods} = \texttt{Total Budget} \quad \Rightarrow \quad \boxed{x + 2y = 60}$$

2.D. Find the optimal ratio of goods x and y the consumer should purchase to maximize their utility.

The optimal ratio of goods x and y can be found using the relative prices of goods (slope of the budget line) and the marginal rate of substitution (slope of the indifference curve).

$$MRS_{xy} = \frac{P_x}{P_y} \quad \Rightarrow \quad \frac{2y}{x} = \frac{1}{2} \quad \Rightarrow \quad \boxed{x = 4y}$$

2.E. Find the optimal bundle that the consumer should purchase to maximize their utility.

Use the budget constraint from `2.C.`, and substitute in the optimal ratio from `2.D.` to find the optimal bundle.

$$\underbrace{x + 2y = 60 \quad \Rightarrow \quad 4y + 2x = 60}_{\because x=4y \text{ from 2.D}} \quad \Rightarrow \quad 6y = 60 \quad \Rightarrow \quad \underbrace{\boxed{y^* = 10} \quad \Rightarrow \quad \boxed{x^* = 40}}_{\because x=4y \text{ from 2.D}}$$

Problem 2. Comparative Statics of the UMP: Income (continued)

2.F. Suppose that the consumer's income increased from \$60 to \$90. Find the optimal bundle that the consumer should purchase to maximize their utility.

The consumer's MRS is unchanged, and neither are the prices. Since the optimal ratio depends on these two factors, is also remains unchanged. Plugging the optimal ratio into the updated budget constraint:

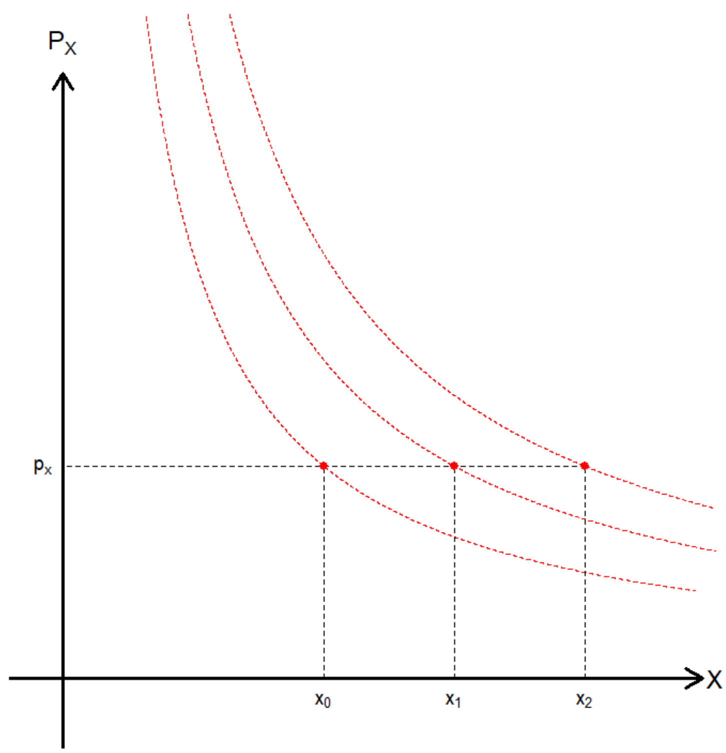
$$x + 2y = 90 \quad \Rightarrow \quad 4y + 2y = 90 \quad \Rightarrow \quad 6y = 90 \quad \Rightarrow \quad \boxed{y^* = 15} \quad \Rightarrow \quad \boxed{x^* = 60}$$

2.G. Suppose that the consumer's income increased from \$90 to \$120. Find the optimal bundle that the consumer should purchase to maximize their utility.

Similar to `2.F`, the consumer's MRS is unchanged, and neither are the prices. Since the optimal ratio depends on these two factors, is also remains unchanged. Plugging the optimal ratio into the updated budget constraint:

$$x + 2y = 120 \quad \Rightarrow \quad 4y + 2y = 120 \quad \Rightarrow \quad 6y = 120 \quad \Rightarrow \quad \boxed{y^* = 20} \quad \Rightarrow \quad \boxed{x^* = 80}$$

2.H. Using your answers from `2.E`, `2.F`, and `2.G`, approximate how the consumer's demand curve reacts to the change in consumers' income in the empty chart below.



Problem 3. Deriving the Demand Curve

Suppose that a consumer's utility function $u(\cdot)$ over two goods x and y is given as:

$$u(x, y) = x^2y^4$$

The consumer's budget is M, and the unit price of good x is P_x, and the unit price of good y is P_y.

3.A. Find the marginal utility of good x and y.

Take the partial derivative of the utility function with respect to x and y.

$$MU_x = \frac{\partial}{\partial x}u(x, y) = \frac{\partial}{\partial x}x^2y^4 = \boxed{2xy^4} \qquad MU_y = \frac{\partial}{\partial y}u(x, y) = \frac{\partial}{\partial y}x^2y^4 = \boxed{4x^2y^3}$$

3.B. Find the marginal rate of substitution between goods x and y.

The marginal rate of substitution is the ratio of the marginal utilities of good x and y.

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{2xy^4}{4x^2y^3} = \boxed{\frac{y}{2x}}$$

3.C. Find the formal expression for the consumer's budget constraint.

The budget constraint is expressed as follows:

$$\texttt{Money Spent on Goods} = \texttt{Total Budget} \quad \Rightarrow \quad \boxed{P_x \cdot x + P_y \cdot y = M}$$

3.D. Find the optimal ratio of goods x and y the consumer should purchase to maximize their utility.

The optimal ratio of goods x and y can be found using the relative prices of goods (slope of the budget line) and the marginal rate of substitution (slope of the indifference curve).

$$MRS_{xy} = \frac{P_x}{P_y} \quad \Rightarrow \quad \frac{y}{2x} = \frac{P_x}{P_y} \quad \Rightarrow \quad \boxed{P_y \cdot y = 2 \cdot P_x \cdot x}$$

3.E. Find the expression for the consumer's demand of good x.

Use the budget constraint from `3.C.`, and substitute in the optimal ratio from `3.D.` to find the optimal bundle.

$$\underbrace{P_x x + P_y y = M \quad \Rightarrow \quad P_x x + 2P_x x = M}_{\because P_y \cdot y = 2 \cdot P_x \cdot x \text{ from 3.D}} \quad \Rightarrow \quad 3P_x x = M \quad \Rightarrow \quad \boxed{x^*(P_x; M) = \frac{1}{3} \cdot \frac{M}{P_x}}$$