Monmouth College

- Name: __________
- Date: __________
- Section: __________

ECON 300: Intermediate Price Theory

Problem Set #3: Suggested Solutions

Fall 2024

Problem 1. Utility Maximization: Cobb-Douglas

Suppose that a consumer's utility function $u(\cdot)$ over two goods x and y is given as:

$$u(x, y) = x^2y^4$$

The consumer's budget is \$60, and the unit price of good x is \$2, and the unit price of good y is \$4.

1.A. Find the marginal utility of good x.

Take the partial derivative of the utility function with respect to x.

$$MU_x = \frac{\partial}{\partial x}u(x, y) = \frac{\partial}{\partial x}x^2y^4 = \boxed{2xy^4}$$

1.B. Find the marginal utility of good y.

Take the partial derivative of the utility function with respect to y.

$$MU_y = \frac{\partial}{\partial y}u(x, y) = \frac{\partial}{\partial y}x^2y^4 = \boxed{4x^2y^3}$$

1.C. Find the marginal rate of substitution between goods x and y.

The marginal rate of substitution is the ratio of the marginal utilities of good x and y.

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{2xy^4}{4x^2y^3} = \boxed{\frac{y}{2x}}$$

1.D. Find the formal expression for the consumer's budget constraint.

The budget constraint is expressed as follows:

$$\texttt{Money Spent on Goods} = \texttt{Total Budget} \quad \Rightarrow \quad \boxed{2x + 4y = 60}$$

1.E. Find the optimal ratio of goods x and y the consumer should purchase to maximize their utility.

The optimal ratio of goods x and y can be found using the relative prices of goods (slope of the budget line) and the marginal rate of substitution (slope of the indifference curve).

$$MRS_{xy} = \frac{P_x}{P_y} \quad \Rightarrow \quad \frac{y}{2x} = \frac{2}{4} \quad \Rightarrow \quad 8y = 8x \quad \Rightarrow \quad \boxed{x = y}$$

Problem 1. Utility Maximization: Cobb-Douglas (continued)

Suppose that a consumer's utility function $u(\cdot)$ over two goods x and y is given as:

$$u(x, y) = x^2 y^4$$

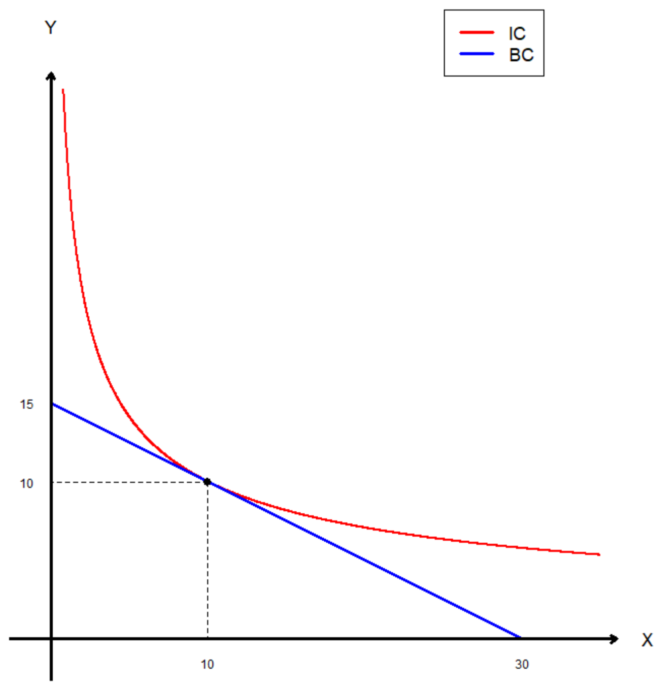
The consumer's budget is \$60, and the unit price of good x is \$2, and the unit price of good y is \$4.

1.F. Find the optimal bundle that the consumer should purchase to maximize their utility.

Use the budget constraint from `1.D.`, and substitute in the optimal ratio from `1.E.` to find the optimal bundle.

$$2x + 4y = 60 \quad \Rightarrow \quad 2x + 4x = 60 \quad \Rightarrow \quad 6x = 60 \quad \Rightarrow \quad \boxed{x^* = 10} \quad \Rightarrow \quad \boxed{y^* = 10}$$

1.G. Plot (A) the consumer's budget constraint, and (B) the consumer's indifference curve that passes through the bundle that maximizes their utility in the empty chart below. The graph need not be to scale, but you must label all of the following items:



- The budget line.
- The indifference curve.
- The x and y intercepts for the budget constraint.
- The utility maximizing bundle of goods.

Problem 2. Utility Maximization: Linear

Suppose that a consumer's utility function $u(\cdot)$ over two goods x and y is given as:

$$u(x, y) = 2x + 3y$$

The consumer's budget is \$30, and the unit price of good x is \$1, and the unit price of good y is \$2.

2.A. Find the marginal utility of good x.

Take the partial derivative of the utility function with respect to x.

$$MU_x = \frac{\partial}{\partial x} u(x, y) = \frac{\partial}{\partial x}(2x + 3y) = \frac{\partial}{\partial x} 2x + \frac{\partial}{\partial x} 3y = 2 + 0 = \boxed{2}$$

2.B. Find the marginal utility of good y.

Take the partial derivative of the utility function with respect to x.

$$MU_y = \frac{\partial}{\partial y} u(x, y) = \frac{\partial}{\partial y}(2x + 3y) = \frac{\partial}{\partial y} 2x + \frac{\partial}{\partial y} 3y = 0 + 3 = \boxed{3}$$

2.C. Find the marginal rate of substitution between goods x and y.

The marginal rate of substitution is the ratio of the marginal utilities of good x and y.

$$MRS_{xy} = \frac{MU_x}{MU_y} = \boxed{\frac{2}{3}}$$

2.D. Find the formal expression for the consumer's budget constraint.

The budget constraint is expressed as follows:

$$\texttt{Money Spent on Goods} = \texttt{Total Budget} \quad \Rightarrow \quad \boxed{x + 2y = 30}$$

2.E. Find the optimal ratio of goods x and y the consumer should purchase to maximize their utility.

We can't use the typical method of equalizing the MRS to relative prices, since both slopes are constant. So, we rely on the "per dollar marginal utility" approach.

$$\texttt{Per \$ } MU_x = \frac{MU_x}{P_x} = \frac{2}{1} > \frac{3}{2} = \frac{MU_y}{P_y} = \texttt{Per \$ } MU_y$$

Good x *always* guarantees a higher marginal utility per dollar spent compared to good y, so the consumer should only purchase good x.

Problem 2. Utility Maximization: Linear (continued)

Suppose that a consumer's utility function $u(\cdot)$ over two goods x and y is given as:

$$u(x, y) = 2x + 3y$$

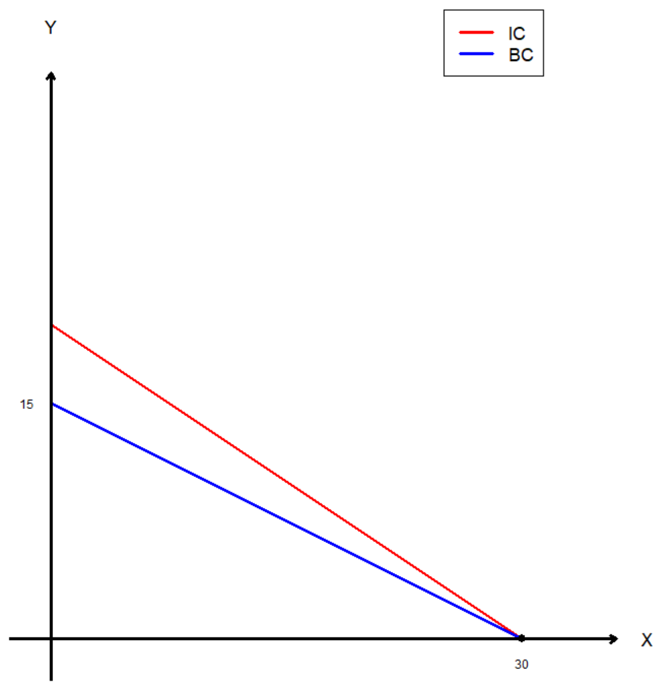
The consumer's budget is \$30, and the unit price of good x is \$1, and the unit price of good y is \$2.

2.F. Find the optimal bundle that the consumer should purchase to maximize their utility.

Based on the answers we derived from `2.E.`, we know that the consumer should spend all of their budget on good x exclusively.

$$\boxed{x^* = 30} \quad \Rightarrow \quad \boxed{y^* = 0}$$

2.G. Plot (A) the consumer's budget constraint, and (B) the consumer's indifference curve that passes through the bundle that maximizes their utility in the empty chart below. The graph need not be to scale, but you must label all of the following items:



- The budget line.
- The indifference curve.
- The x and y intercepts for the budget constraint.
- The utility maximizing bundle of goods.

Problem 3. Utility Maximization: Leontief

Suppose that a consumer's utility function $u(\cdot)$ over two goods x and y is given as:

$$u(x,y) = min\{2x, y\}$$

The consumer's budget is \$60, and the unit price of good x is \$5, and the unit price of good y is \$5.

3.A. Find the marginal utility of good x.

The *min* function is not differentiable, so our typical definition of marginal utility does not apply.

3.B. Find the marginal utility of good y.

The *min* function is not differentiable, so our typical definition of marginal utility does not apply.

3.C. Find the marginal rate of substitution between goods x and y.

Since marginal utility is not defined, we cannot define the marginal rate of substitution for the Leontief utility function. Recall that the Leontief utility function is used specifically when the two goods are perfect *complements*, so there is absolutely no substitutability between x and y. Therefore, it makes sense that the marginal rate of *substitution* is not defined.

3.D. Find the formal expression for the consumer's budget constraint.

The budget constraint is expressed as follows:

$$\texttt{Money Spent on Goods} = \texttt{Total Budget} \quad \Rightarrow \quad \boxed{5x + 5y = 60}$$

3.E. Find the optimal ratio of goods x and y the consumer should purchase to maximize their utility.

The optimal ratio is to always keep the arguments of the *min* function equal.

$$\boxed{2x = y}$$

If we allow $2x > y$, then there will be some amount of good x that is purchased, that does not contribute to the utility of the consumer. If we allow $2x < y$, then there will be some amount of good y that is consumed, which does not contribute to the consumer's utility. Therefore, the only logical choice is to allow $2x = y$.

Problem 3. Utility Maximization: Leontief (continued)

Suppose that a consumer's utility function $u(\cdot)$ over two goods x and y is given as:

$$u(x, y) = min\{2x, y\}$$

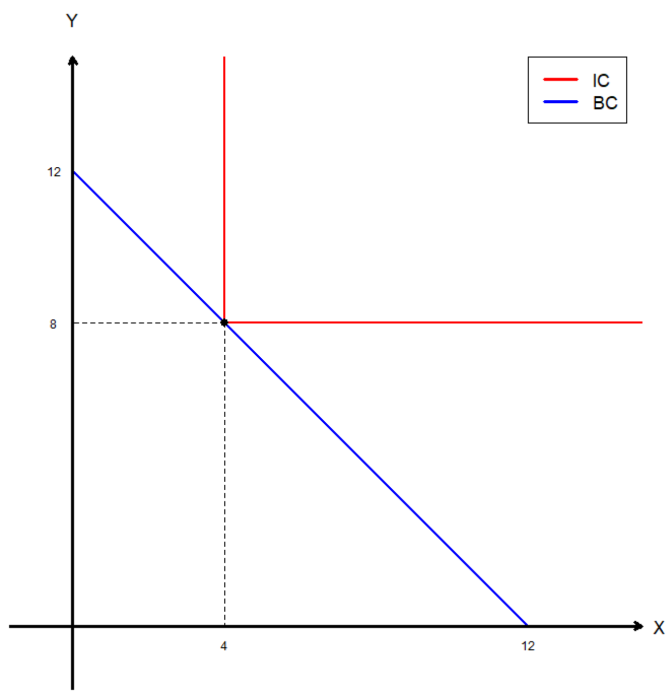
The consumer's budget is \$60, and the unit price of good x is \$5, and the unit price of good y is \$5.

3.F. Find the optimal bundle that the consumer should purchase to maximize their utility.

Use the budget constraint from `3.D.` and the optimal ratio from `3.E.` to find the optimal bundle.

$$5x + 5y = 60 \quad \Rightarrow \quad 5x + 5(2x) = 60 \quad \Rightarrow \quad 15x = 60 \quad \Rightarrow \quad \boxed{x^* = 4} \quad \Rightarrow \quad \boxed{y^* = 8}$$

3.G. Plot (A) the consumer's budget constraint, and (B) the consumer's indifference curve that passes through the bundle that maximizes their utility in the empty chart below. The graph need not be to scale, but you must label all of the following items:



- The budget line.
- The indifference curve.
- The x and y intercepts for the budget constraint.
- The utility maximizing bundle of goods.

- Score: ______________________
- Extra Credit: ______________________