- Name: ____________________
- Date: ____________________
- Section: ____________________

ECON 300: Intermediate Price Theory

Problem Set #3: Suggested Solutions

INSTRUCTIONS:

- This problem set is not graded.

Problem 1. The Budget Constraint

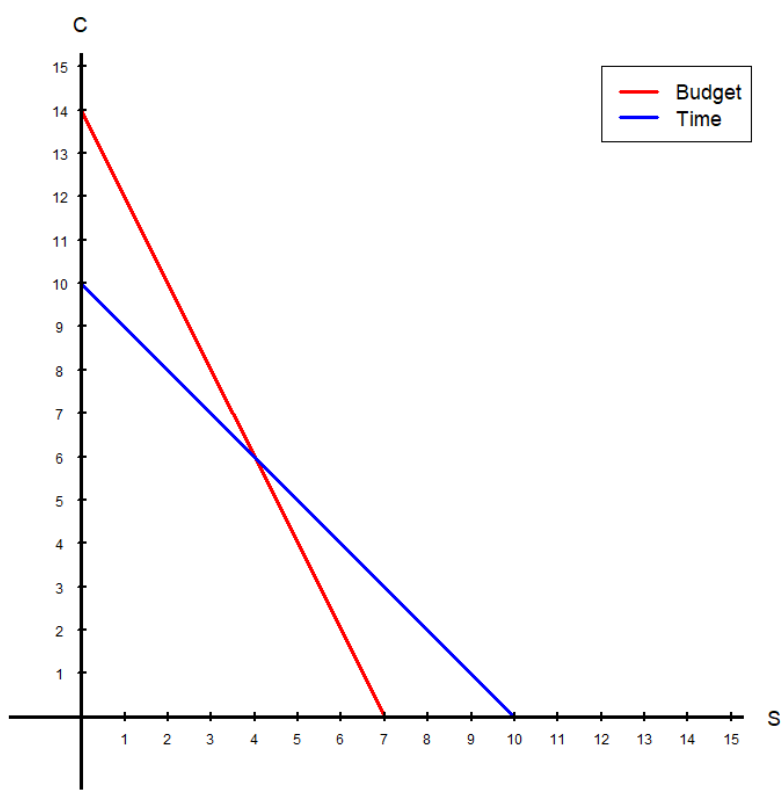
Suppose that you are headed to the Maldives for spring break. The activities that are available on the resort hotel are scuba diving (s) and cooking lessons (c). Your budget is $700 for the day, and an hour of scuba diving costs $100, and an hour of cooking lessons costs $50. You have one day on the resort, so not only do you have to spend your money wisely, you must allocate your time wisely. You have 10 hours that you can spend on any combination of the two activities.

1.A. Express your budget constraint and time constraint as two separate equations.

- The Budget Constraint: $\boxed{100s + 50c = 700}$

- The Time Constraint: $\boxed{s + c = 10}$

1.B. Plot (overlay) the consumer's budget and time constraint in the diagram below.



1.C. Is either the budget constaint or the time constraint redundant?

Solution:

According to our answers in the previous problem 1.B., there are some combinations of bundles that we lack time to enjoy but are affordable, and some combinations that are affordable while not having enough time to enjoy the activities. Therefore, both contraints are relevant in decision making.

1.D. Suppose that due to a happy accident in the hotel's management system, you get to stay at the resort free of charge for an extra day. The total amount of time you have is now 15 hours. Is either the budget or time constraint redundant?

Solution:

If our time constraint is increased to a total of 15 hours, we have enough time to enjoy any bundle that is affordable. Therefore, the time constraint becomes redundant.

1.E. What is the slope of the budget (money) constraint?

Solution:

The slope of the budget constraint can be calculated using two methods; the ratio of market prices, or the rise-over-run formula.

$$\texttt{Slope} = \frac{P_s}{P_c} = \frac{100}{50} = 2, \quad \texttt{or} \quad \texttt{Slope} = \frac{Rise}{Run} = \frac{14}{7} = 2$$

1.F. Complete the following statement based on your answer in Question 1.E.

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"In order to attend an extra hour of SCUBA diving,
 you must give up on  2  hours of cooking lessons."
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Problem 2. Calculating Marginal Utility

Find the expression for the marginal utility of goods x and y for the following utility functions:

2.A. $u(x, y) = 3x$

- $MU_x = \frac{\partial}{\partial x} 3x = 3 \cdot 1 \cdot x^{1-1} = \boxed{3}$

- $MU_y = \frac{\partial}{\partial y} 3x = \boxed{0}$

2.B. $u(x, y) = 10$

- $MU_x = \frac{\partial}{\partial x} 10 = \boxed{0}$

- $MU_y = \frac{\partial}{\partial y} 10 = \boxed{0}$

2.C. $u(x, y) = 3xy^3$

- $MU_x = \frac{\partial}{\partial x} 3xy^3 = 3y^3 \cdot 1 \cdot x^{1-1} = \boxed{3y^3}$

- $MU_y = \frac{\partial}{\partial y} 3xy^3 = 3x \cdot 3 \cdot y^{3-1} = \boxed{9xy^2}$

2.D. $u(x, y) = x^3y^5$

- $MU_x = \dfrac{\partial}{\partial x}x^3y^5 = y^5 \cdot 3 \cdot x^{3-1} = \boxed{3x^2y^5}$

- $MU_y = \dfrac{\partial}{\partial y}x^3y^5 = x^3 \cdot 5 \cdot y^{5-1} = \boxed{5x^3y^4}$

2.E. $u(x, y) = 2x + y$

- $MU_x = \dfrac{\partial}{\partial x}(2x + y) = \dfrac{\partial}{\partial x}2x + \dfrac{\partial}{\partial x}y = 2 + 0 = \boxed{2}$

- $MU_y = \dfrac{\partial}{\partial y}(2x + y) = \dfrac{\partial}{\partial y}2x + \dfrac{\partial}{\partial y}y = 0 + 1 = \boxed{1}$

2.F. $u(x, y) = 5x + 3y$

- $MU_x = \dfrac{\partial}{\partial x}(5x + 3y) = \dfrac{\partial}{\partial x}5x + \dfrac{\partial}{\partial x}3y = 5 + 0 = \boxed{5}$

- $MU_y = \dfrac{\partial}{\partial y}(5x + 3y) = \dfrac{\partial}{\partial y}5x + \dfrac{\partial}{\partial y}3y = 0 + 3 = \boxed{3}$

Problem 3. Marginal Analysis and Utility Maximization Concepts

3.A. Suppose for this specific problem, goods are sold in discrete units of 1. Fill out the bottom row with the marginal utility values.

Quantity of x	1	2	3	4	5	6	7	8	9
$u(x)$	200	360	420	470	495	510	520	527	530
MU_x	200	160	60	50	25	15	10	7	3

3.B Complete the statement below:

"According to the law of diminishing marginal utility , today's 1st apple that I consume should give me a greater level of utility compared to my 2nd apple of the day."

3.C What is the definiton of the marginal rate of substitution between goods x and y?

- $MRS_{xy} = \dfrac{MU_x}{MU_y}$

3.D Complete the statement below:

"The marginal rate of substitution MRS_{xy} is the maximum amount of good y that the consumer is willing to give up for 1 extra unit of good x."

3.E In your own words, explain what the consumer should do if $MRS_{xy} > \frac{P_x}{P_y}$.

Solution:

MRS represents the maximum amount of good y that the consumer is "willing to pay" for an extra unit of good x. Meanwhile, P_x/P_y represents the amount of good y that the consumer "has to pay" for one extra unit of good x. So, if $MRS > P_x/P_y$, the consumer is willing to pay more than what is required to get an extra unit of good x, and the consumer should move to a bundle with more x and less y.

3.F Show that the following two conditions are equivalent (start from the equality on the left, and transform it to the equality on the right).

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y} \quad \Leftrightarrow \quad \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

Solution:

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y} \quad \Rightarrow \quad \frac{MU_x}{MU_y} \cdot MU_y = \frac{P_x}{P_y} \cdot MU_y \qquad \because \text{Multiply } MU_y \text{ to both sides}$$

$$\Rightarrow \quad \frac{MU_x}{\cancel{MU_y}} \cdot \cancel{MU_y} = \frac{P_x}{P_y} \cdot MU_y \qquad \because \text{Cancel out } MU_y \text{ on the left}$$

$$\Rightarrow \quad MU_x = \frac{MU_y \cdot P_x}{P_y}$$

$$\Rightarrow \quad MU_x \cdot \left(\frac{1}{P_x}\right) = \frac{MU_y \cdot P_x}{P_y} \cdot \left(\frac{1}{P_x}\right) \qquad \because \text{Multiply } 1/P_x \text{ to both sides}$$

$$\Rightarrow \quad MU_x \cdot \left(\frac{1}{P_x}\right) = \frac{MU_y \cdot \cancel{P_x}}{P_y} \cdot \cancel{\left(\frac{1}{P_x}\right)} \qquad \because \text{Cancel out } P_x \text{ on the right}$$

$$\Rightarrow \quad \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

Problem 4. Utility Maximization and Substitution Effects

Suppose that the consumer is participating in a market with goods x and y. Each unit of good x costs \$10, each unit of good y costs \$15, and the consumer's budget is \$150. The consumer's utility derives from the following utility function:

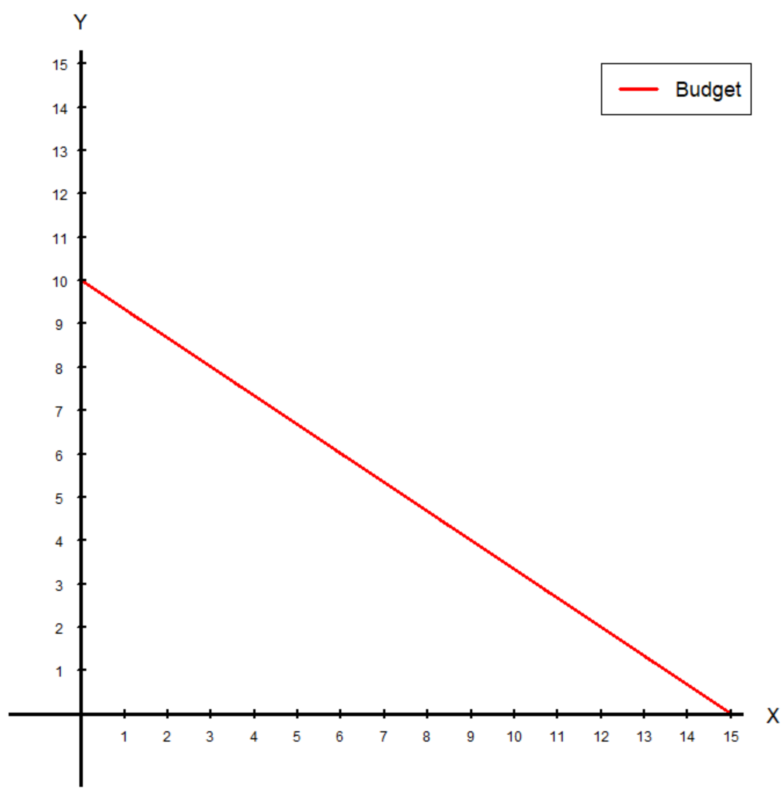
$$u(x, y) = 4x^3y^2$$

4.A. Express the consumer's budget constraint as a mathematical equation.

Solution:

$$10x + 15y = 150$$

4.B. Plot the consumer's budget line in the diagram below.



4.C. Find the marginal utility of good x and y:

- $MU_x = \frac{\partial}{\partial x} 4x^3y^2 = 4y^2 \cdot 3 \cdot x^{3-1} = \boxed{12x^2y^2}$

- $MU_y = \frac{\partial}{\partial y} 4x^3y^2 = 4x^3 \cdot 2 \cdot y^{2-1} = \boxed{8x^3y}$

4.D. Find the expression for the marginal rate of substitution.

Solution:

$$
\begin{aligned}
MRS = \frac{MU_x}{MU_y} &= \frac{12x^2y^2}{8x^3y} \\
&= \frac{\cancel{12}^{3}x^2y^2}{\cancel{8}^{2}x^3y} && \because \frac{12}{8} = \frac{3}{2} \\
&= \frac{3\cancel{x^2}y^2}{2x^{\cancel{3}}y} && \because \text{Cancel out } x^2 \\
&= \frac{3y^{\cancel{2}}}{2x\cancel{y}} && \because \text{Cancel out } y \\
&= \boxed{\frac{3y}{2x}}
\end{aligned}
$$

4.E. Find the optimal ratio between good x and y.

Solution:

$$MRS = \frac{P_x}{P_y} \quad \Rightarrow \quad \frac{3y}{2x} = \frac{10}{15} \quad \Rightarrow \quad 20x = 45y \quad \Rightarrow \quad \boxed{x = \frac{9}{4}y}$$

4.F. Find the optimal bundle x_0^* and y_0^*:

Solution:

Plug the optimal ratio we found in 4.E. into the budget constraint we found in 4.A.

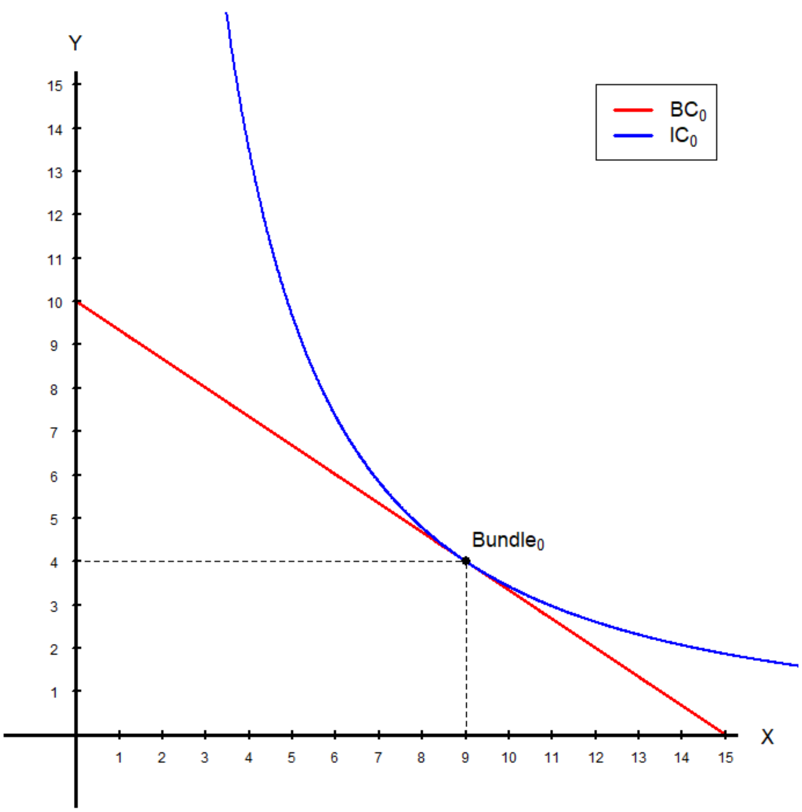
$$10x + 15y = 150 \quad \Rightarrow \quad 10\left(\frac{9}{4}y\right) + 15y = 150 \quad \Rightarrow \quad \frac{150}{4}y = 150 \quad \Rightarrow \quad \boxed{y_0^* = 4}$$

Using the optimal ratio equation from 4.E. we find:

$$x = \frac{9}{4}y \quad \Rightarrow \quad x_0^* = \frac{9}{4}y_0^* \quad \Rightarrow \quad x_0^* = \frac{9}{4}\cdot 4 \quad \Rightarrow \quad \boxed{x_0^* = 9}$$

4.G. Plot and label the following items:

1. The consumer's budget line (from 4.B),
2. The optimal bundle (x_0^*, y_0^*)
3. The indifference curve for the utility maximizing consumer.



4.H. Suppose that the price of good x increased from the original \$10 to \$15, while no other variable changes. What would be the new expression for the budget constraint?

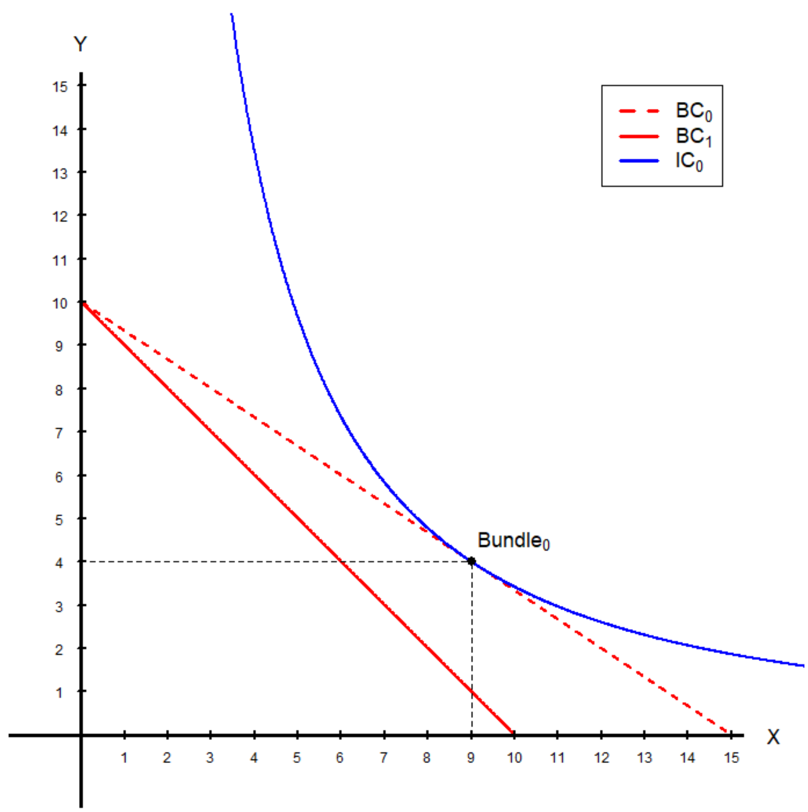
Solution:

$$15x + 15y = 150$$

4.I. Plot and label the following items:

1. The consumer's original budget line (from 4.B),
2. The optimal bundle (x_0^*, y_0^*)
3. The indifference curve for the utility maximizing consumer.
4. The new budget line

Hint: The indifference curve does not have to be exactly to scale.



4.J. Under the updated price of $P_x^1 = 15$, what is the updated "optimal ratio" of goods x and y?
Hint: The MRS remains constant, but something else changed...

Solution:

$$MRS = \frac{P_x}{P_y} \quad \Rightarrow \quad \frac{3y}{2x} = \frac{15}{15} \quad \Rightarrow \quad 30x = 45y \quad \Rightarrow \quad \boxed{x = \frac{3}{2}y}$$

4.K Under the updated price of $P_x^1 = 15$, what is the new optimal bundle (x_1^*, y_1^*)?

Solution:

Plug the optimal ratio we found in 4.J. into the budget constraint we found in 4.H.

$$15x + 15y = 150 \quad \Rightarrow \quad 15\left(\frac{3}{2}y\right) + 15y = 150 \quad \Rightarrow \quad \frac{75}{2}y = 150 \quad \Rightarrow \quad \boxed{y_1^* = 4}$$

Using the optimal ratio equation from 4.J. we find:

$$x = \frac{3}{2}y \quad \Rightarrow \quad x_1^* = \frac{3}{2}y_1^* \quad \Rightarrow \quad x_1^* = \frac{3}{2} \cdot 4 \quad \Rightarrow \quad \boxed{x_1^* = 6}$$

4.L Is good x an ordinary good? Why?
Hint: Compare x_0^* to x_1^*, and consider how price changed.

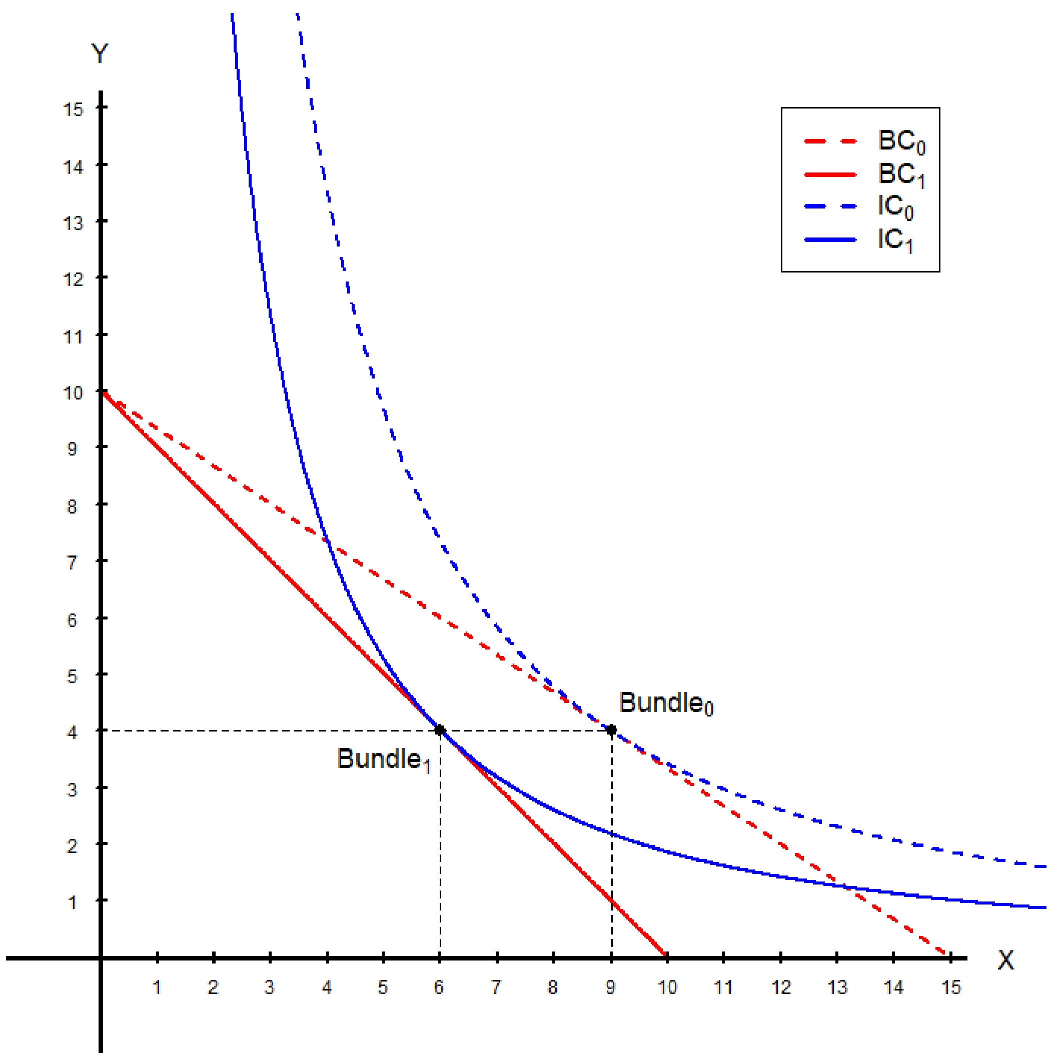
Solution:

Price of good x increased from $P_x^0 = 10$ to $P_x^1 = 15$, and the optimal quantity of good x decreased from $x_0^* = 9$ to $x_1^* = 6$, so good x is an ordinary good.

4.M. Plot and label the following items:

1. The consumer's original budget line (from 4.B),
2. The original optimal bundle (x_0^*, y_0^*)
3. The original indifference curve passing through (x_0^*, y_0^*)
4. The new budget line
5. The updated optimal bundle (x_1^*, y_1^*)
6. The new indifference curve passing through (x_1^*, y_1^*)

Hint: The indifference curve does not have to be exactly to scale.



4.N. In your own words, describe the following terms:

- Price Effect

  Solution:

  Quantity change of a good induced by a change in its market price.

- Substitution Effect

  Solution:

  The portion of the price effect that accounts for the pure effect of the change in relative prices, isolated away from any changes due to shifts in purchasing power.

- Income Effect

  Solution:

  The portion of the price effect that accounts for the pure effect of the change in purchasing power, isolated away from any changes due to changes in the relative prices.

4.O. Select the options that complete the statement correctly:

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"The Hicksian substitution effect can be found by
 finding the budget line that [     A     ], while
 also [     B     ]."
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Q1: What is the correct phrase to populate $\boxed{A}$?

(a) Is parallel to the new budget line

(b) Is parallel to the old budget line

Q2: What is the correct phrase to populate $\boxed{B}$?

(a) Passing through the old optimal bundle

(b) Passing through the new optimal bundle

(c) Being tangent to the old indifference curve

(d) Being tangent to the new indifference curve

4.P. Plot and label the following items:

1. The consumer's original budget line (from 4.B),
2. The original optimal bundle (x_0^*, y_0^*)
3. The original indifference curve passing through (x_0^*, y_0^*)
4. The new budget line
5. The updated optimal bundle (x_1^*, y_1^*)
6. The new indifference curve passing through (x_1^*, y_1^*)
7. The Hicksian Substitution Effect ("budget line" and bundle)