Monmouth College

- Name: ______
- Date: ______
- Section: ______

ECON 300: Intermediate Price Theory

Problem Set #2: Suggested Solutions

INSTRUCTIONS:

- This problem set is not graded.

Problem 1. The Budget Constraint

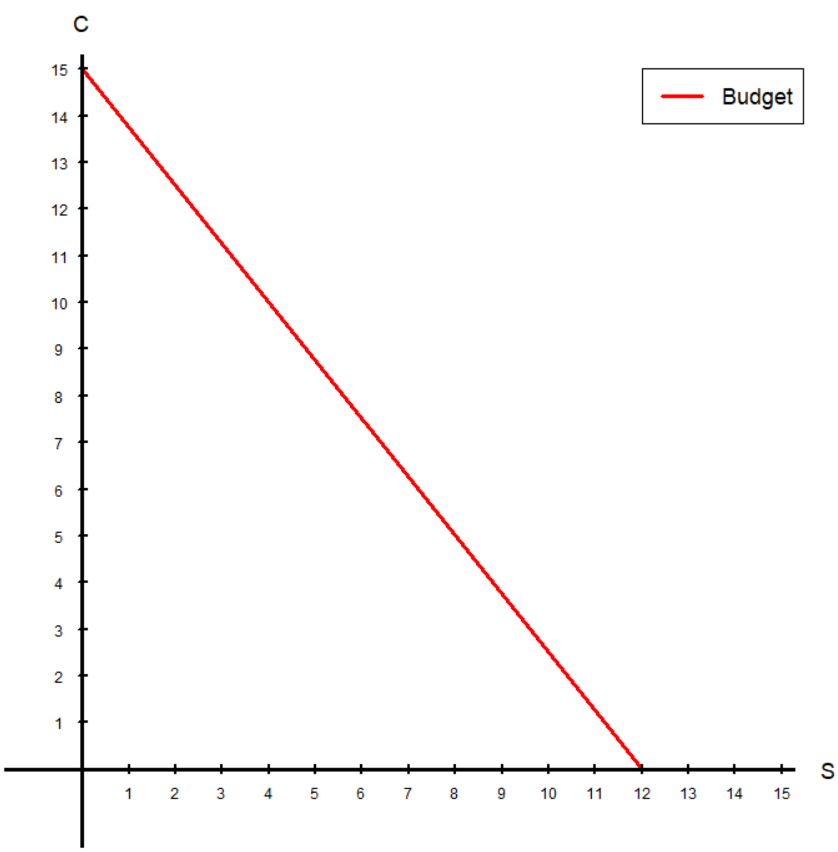
Suppose that you are preparing for a trip abroad with a budget of $60. You are purchasing and packing toiletries, and the two items you need are shampoo (s) and conditioner (c). Each ounce of shampoo costs $5, and each ounce of a conditioner costs $4. You have no other liquids than the amount of shampoo and conditioner you purchased, but the TSA will allow up to 10 ounces of liquids onboard an aircraft.

1.A. Express the consumer's budget constraint and TSA constraint as two separate equations.

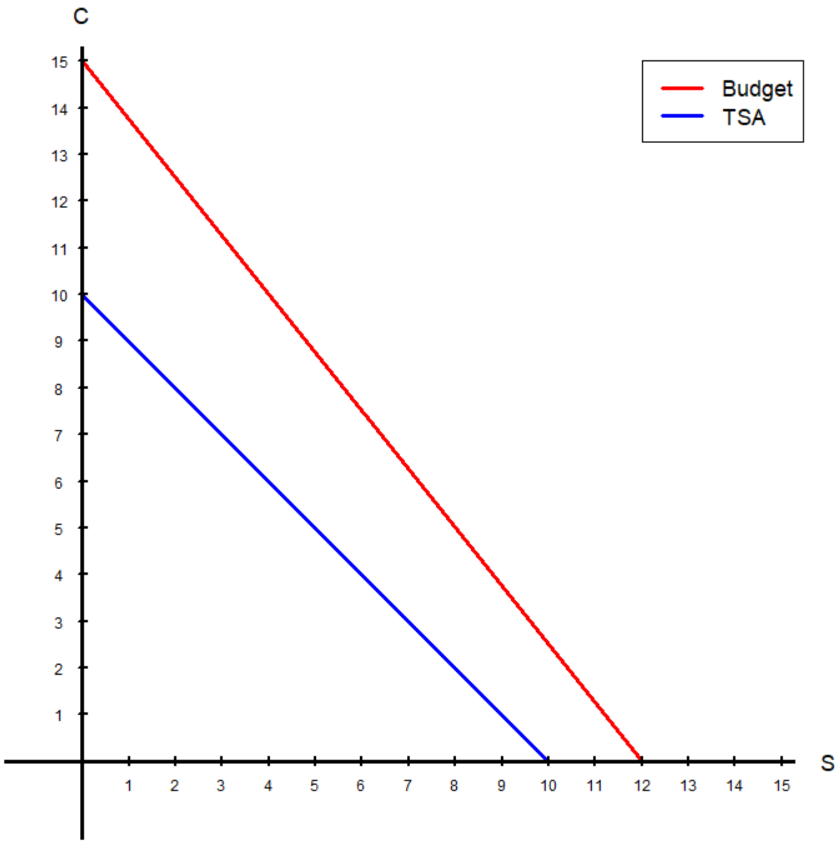
- The Budget Constraint: $\boxed{5s + 4c = 60}$

- The TSA Constraint: $\boxed{s + c = 10}$

1.B. Plot the consumer's budget constraint in the diagram below.



1.C. Plot the consumer's budget constraint and TSA constraint in the diagram below.



1.D. Is either the budget constaint or the TSA constraint redundant? If so, which one is the *binding* constraint?

Solution:

Any bundle that is affordable (below the budget line) also passes the TSA's requirements (below the TSA line). Therefore, the budget line is redundant for the consumer, and the *binding* constraint is the TSA constraint.

Problem 2. The Utility Maximization Problem

Suppose that you are the consumer from **Problem 1.** Your utility function over shampoo (s) and conditioner (c) is defined as:

$$u(s, c) = s^2 c$$

2.A. Find the formula for the marginal utility of shampoo and the marginal utility of conditioner.

- $MU_s = \frac{\partial}{\partial s} s^2 c = \boxed{2sc}$
- $MU_c = \frac{\partial}{\partial c} s^2 c = \boxed{s^2}$

2.B. Find the formula for the marginal rate of substitution and the *price ratio* (slope of the constraint).

- $MRS_{s,c} = \frac{MU_s}{MU_c} = \frac{2sc}{s^2} = \boxed{\frac{2c}{s}}$
- "Price" Ratio = $\boxed{1}$

2.C. Find the optimal units of shampoo and conditioner that will maximize this consumer's utility.

Solution:

Set the MRS equal to the "Price" ratio, and find the optimal ratio of shampoo to conditioner:

$$\frac{2c}{s} = 1 \;\Rightarrow\; 2c = s$$

Then use the TSA constraint, which is the *binding* constraint, to determine the optimal bundle:

$$s + c = 10 \;\Rightarrow\; (2c) + c = 10 \;\Rightarrow\; \boxed{c^* = \frac{10}{3}} \;\Rightarrow\; \boxed{s^* = \frac{20}{3}}$$

Problem 3. Deriving the Engel Curve

Suppose that the consumer is participating in a market consisting of goods x and y. The price of good x is 2, the price of good y is 1. The consumer's utility function is given as follows:

$$u(x, y) = x + 2y$$

3.A. If the consumer's income is $M_0 = 2$, what is the consumer's optimal consumption bundle?

Solution:

Since the per dollar utility is greater for good y, and good x and y are perfect substitutes, the consumer spends all of their budget on good y:

$$\boxed{y^*(P_x, P_y, 2) = 2}$$

3.B. If the consumer's income is $M_1 = 6$, what is the consumer's optimal consumption bundle?

Solution:

$$\boxed{y^*(P_x, P_y, 6) = 6}$$

3.C. If the consumer's income is $M_2 = 10$, what is the consumer's optimal consumption bundle?

Solution:

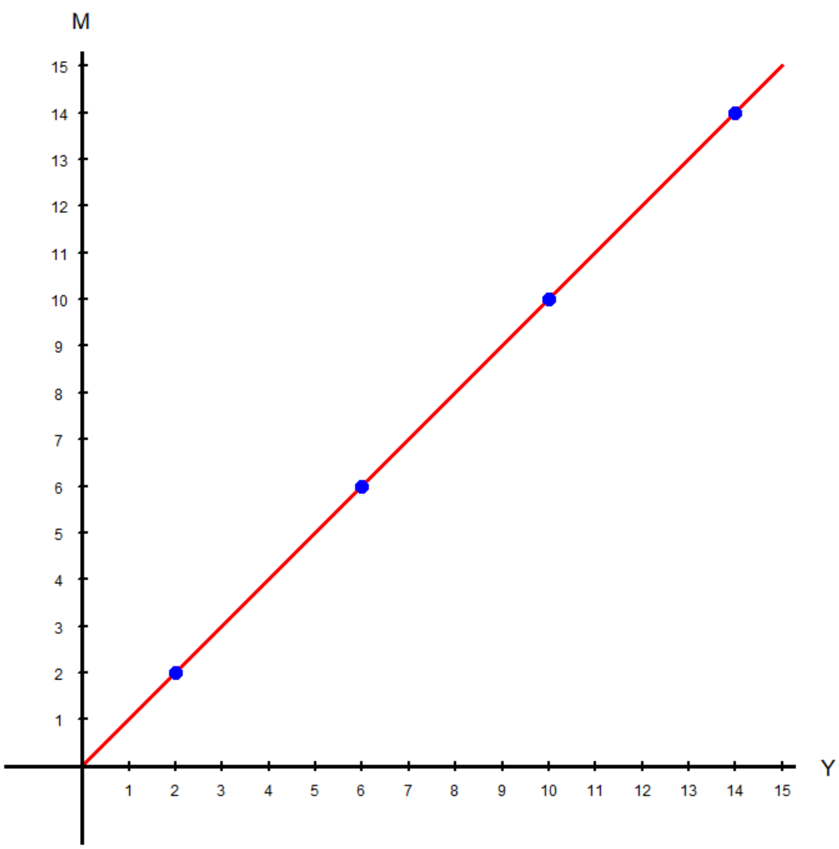
$$\boxed{y^*(P_x, P_y, 10) = 10}$$

3.D. If the consumer's income is $M_3 = 14$, what is the consumer's optimal consumption bundle?

Solution:

$$\boxed{y^*(P_x, P_y, 14) = 14}$$

3.E. Plot the consumer's Engel curve for good y in the diagram below.



3.F. Is good y a Normal good or an Inferior good? Why?

Solution:

Good y is a normal good in this situation, as the quantity demanded of y increases as the consumer's income increases.

Problem 4. Deriving the Walrasian Demand Curve

Suppose that the consumer is participating in a market consisting of goods x and y. The price of good x is P_x, the price of good y is P_y, and the consumer's income is M. The consumer's utility function is given as follows:

$$u(x, y) = 4xy^3$$

4.A. Find the formula for the marginal utility of x and the marginal utility of y.

- $MU_x = \frac{\partial}{\partial x} 4xy^3 = \boxed{4y^3}$
- $MU_y = \frac{\partial}{\partial y} 4xy^3 = \boxed{12xy^2}$

4.B. Find the optimal ratio of goods x and y for this consumer.

Solution:

$$MRS = \frac{P_x}{P_y} \Rightarrow \frac{4y^3}{12xy^2} = \frac{P_x}{P_y} \Rightarrow \frac{y}{3x} = \frac{P_x}{P_y} \Rightarrow \boxed{P_y y = 3P_x x}$$

4.C. Find the demand function for good x.

Solution:

Insert the optimal ratio into the budget constraint:

$$P_x x + P_y y = M \Rightarrow P_x x + (3P_x x) = M \Rightarrow 4P_x x = M \Rightarrow \boxed{x^*(P_x, P_y, M) = \frac{M}{4P_x}}$$

4.D. Find the demand function for good y.

Solution:

Insert the optimal ratio into the budget constraint:

$$P_x x + P_y y = M \Rightarrow \left(\frac{1}{3} P_y y\right) + P_y y = M \Rightarrow \frac{4}{3} P_y y = M \Rightarrow \boxed{y^*(P_x, P_y, M) = \frac{3M}{4P_x}}$$

Problem 5. The Income and Substitution Effects

Suppose that the consumer is in a market with coffee (c) and bagels (b). The price of each cup of coffee is $P_c = \$4$, the price of each bagel is $P_b = \$2$, and the consumer has \$32 as their income. The consumer's utility function over coffee and bagles is defined as:

$$u(c, b) = c^3 b$$

5.A. Find the bundle of coffee and bagels that will maximize the consumer's utility.

Solution:

First we find the optimal ratio by setting the marginal rate of substitution to the price ratio:

$$\frac{3c^2 b}{c^3} = \frac{4}{2} \;\Rightarrow\; \frac{3b}{c} = 2 \;\Rightarrow\; 2c = 3b \;\Rightarrow\; c = \frac{3}{2}b$$

Insert the optimal ratio into the budget constraint:

$$4c + 2b = 32 \;\Rightarrow\; 4\left(\frac{3}{2}b\right) + 2b = 32 \;\Rightarrow\; 8b = 32 \;\Rightarrow\; \boxed{b_0^* = 4} \;\Rightarrow\; \boxed{c_0^* = 6}$$

5.B. Suppose that the price of coffee falls to $P_c' = \$2$. Find the optimal bundle for the consumer.

Solution:

First we find the optimal ratio by setting the marginal rate of substitution to the price ratio:

$$\frac{3c^2 b}{c^3} = \frac{2}{2} \;\Rightarrow\; \frac{3b}{c} = 1 \;\Rightarrow\; c = 3b$$

Insert the optimal ratio into the budget constraint:

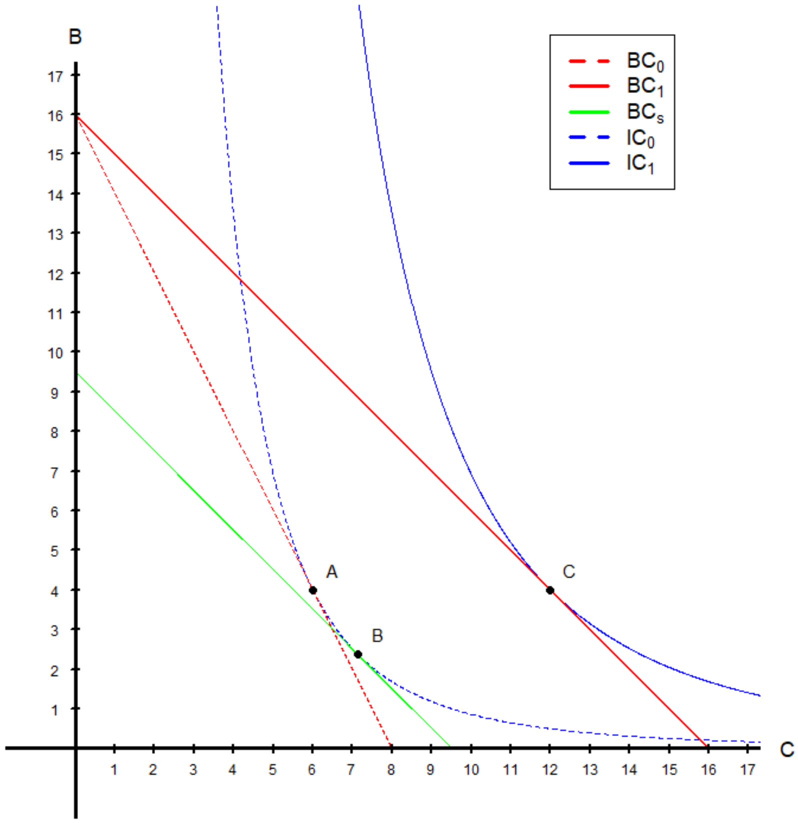
$$2c + 2b = 32 \;\Rightarrow\; 2\,(3b) + 2b = 32 \;\Rightarrow\; 8b = 32 \;\Rightarrow\; \boxed{b_1^* = 4} \;\Rightarrow\; \boxed{c_1^* = 12}$$

5.C. Is coffee an Ordinary good or a Giffen good in this situation? Why?

Solution:

As price decreased from \$4 to \$2, we see that the quantity demanded for coffee increases from 6 to 12. Therefore, coffee is an Ordinary good.

5.D. Plot this change in the diagram below, showing the income and substitution effects.



- A→B: Substitution Effect
- B→C: Income Effect

5.E. (ADVANCED) Calculate the income and substitution effects of coffee.

Solution:

This problem boils down to finding the bundle B in our answer for the previous question. Note that the (Hicksian) substitution effect is found by...

- Keeping the utility constant at the pre-change level, and
- Finding what the optimal bundle would have been under the new price regime.

Thus, the first task is to find the pre-change utility level:

$$u(c_0^*, b_0^*) = 6^3 \cdot 4 = 864$$

Now, recall that the new optimal ratio under the new price regime was $c = 3b$.[1] We can now find the units of coffee and bagels in bundle B, (c_s^*, b_s^*) as follows:

$$c^3 b = 864 \;\Rightarrow\; (3b)^3 b = 864 \;\Rightarrow\; 27b^4 = 864 \;\Rightarrow\; b^4 = 32 \;\Rightarrow\; b_s^* = 32^{\frac{1}{4}} \simeq 2.378$$

Use the optimal ratio $c = 3b$ again to find the units of coffee in bundle B.

$$c = 3b \;\Rightarrow\; c_s^* = 3\left(32^{\frac{1}{4}}\right) \simeq 7.135$$

Thus we can conclude as follows:

- Substitution Effect: $c_0^* \to c_s^* : 6 \to 7.135$
- Income Effect: $c_s^* \to c_1^* : 7.135 \to 12$

[1] Read the solutions to problem 5.B.