Monmouth COLLEGE

- Name: ______
- Date: ______
- Section: ______

ECON 300: Intermediate Price Theory

Problem Set #1: Suggested Solutions

INSTRUCTIONS:

- This problem set is not graded.

Problem 1. The Budget Constraint

Suppose that the consumer is participating in a market consisting of goods x and y. Good x costs \$3, and good y costs \$2 for each unit. The consumer has \$30 as their income, that they will exhaust on consuming goods x and y.

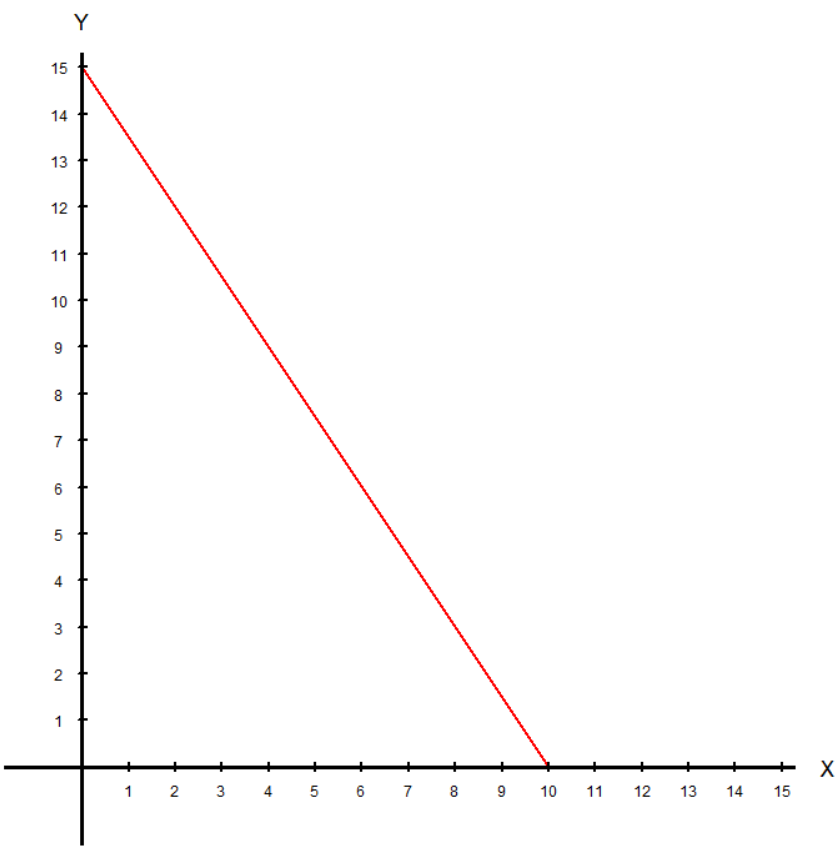
1.A. Express the consumer's budget constraint as a mathematical equation.

Solution:

The budget constraint should be "expenditure" is equal to "income." So the consumer's budget constraint in this case should be:

$$\underbrace{3x + 2y}_{P_x x + P_y y} = \underbrace{30}_{M}$$

1.B. Plot the consumer's budget line in the diagram below.



1.C. Calculate the slope of the budget line.

Solution:

The slope of the budget line can be calculated using two different methods. We can use the relative prices, or we can use the rise-over-run formula:

$$\texttt{Slope} = -\frac{P_x}{P_y} = -\frac{3}{2}, \quad \texttt{Slope} = \frac{\texttt{Rise}}{\texttt{Run}} = \frac{15}{-10} = \boxed{-\frac{3}{2}}$$

1.D. Assuming the consumer is currently consuming bundle $(x, y) = (6, 10)$. Finish the statement:

"The consumer must give up $\frac{3}{2}$ units of good y to purchase 1 extra unit of good x."

1.E. In your own words, define the following terms:

- The Budget Set

  Solution:

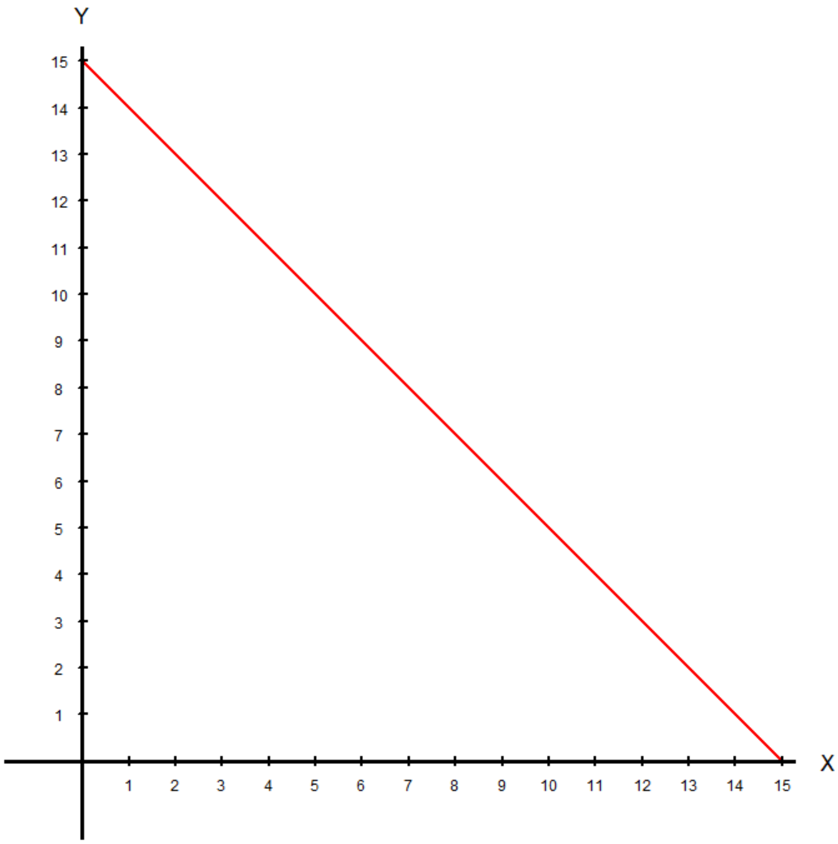
  The budget set is the set of all bundles that are feasible for the consumer. These bundles will cost less than the consumer's income, and can be graphically represented as the area below the budget line.

- The Budget Line

  Solution:

  The budget line is (typically) the collection of all bundles that cost exactly the same as the consumer's income.

1.G. Assuming the all other market conditions are identical to the beginning of **Problem 1**, plot the consumer's budget line if the market price of good x falls to \$2.



Problem 2. Utility and Indifference Curves

Suppose that the consumer is participating in a market consisting of goods x and y. The consumer's utility function is given as:

$$u(x, y) = xy$$

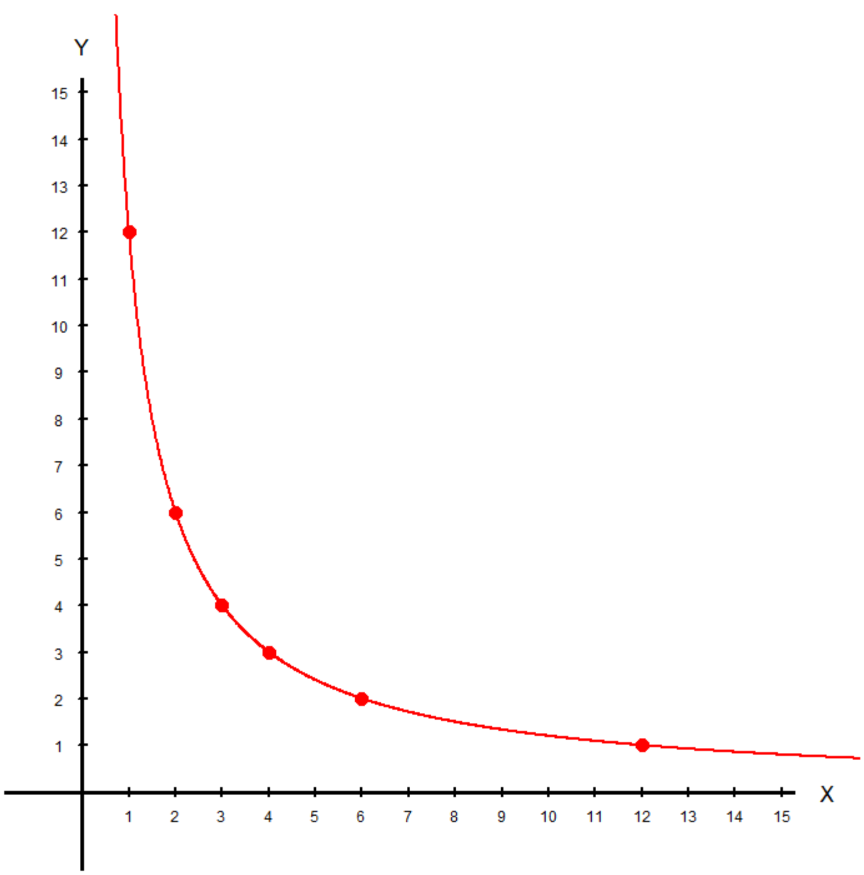
2.A. Find the formula for the marginal utility of good x.

Solution:

The marginal utility of good x is the partial derivative of the utility function with respect to x:

$$MU_x = \frac{\partial}{\partial x} u(x, y) = \frac{\partial}{\partial x} xy = \boxed{y}$$

2.B. Plot 4 bundles that will provide the consumer with 12 utility, and approximate the indifference curve representing 12 utility.



2.C. Find the expression for the marginal rate of substitution (MRS).

Solution:

The marginal rate of substitution is the ratio of the marginal utilities:

$$MRS_{xy} = -\frac{MU_x}{MU_y} = -\frac{\frac{\partial}{\partial x}u(x,y)}{\frac{\partial}{\partial y}u(x,y)} = -\frac{\frac{\partial}{\partial x}xy}{\frac{\partial}{\partial y}xy} = \boxed{-\frac{y}{x}}$$

2.D. Calculate the MRS for the consumer that is consuming bundle $(x, y) = (4, 3)$.

Solution:

Insert the values of x and y into the expression for the MRS we found in part 2.C.:

$$MRS_{xy} = -\frac{y}{x} = \boxed{-\frac{3}{4}}$$

2.E. Assuming the consumer is currently consuming bundle $(x, y) = (4, 3)$. Finish the statement:

"The consumer is willing to give up $\frac{3}{4}$ units good y to consume 1 extra unit of good x."

2.F. If we believe that the goods x and y are perfect substitutes, is the current utility function an appropriate choice? If not, what would be a suitable utility function?

Solution:

No, if the goods x and y are perfect substitutes, we should be using the linear utility function to represent the consumer's utility.

Problem 3. The Utility Maximization Problem

Suppose that the consumer is participating in a market consisting of goods x and y. Good x costs \$3, and good y costs \$2 for each unit. The consumer has \$30 as their income, that they will exhaust on consuming goods x and y. The consumer's utility function is given as:

$$u(x, y) = 2x^2y$$

3.A. Find the marginal utility of x.

Solution:

The marginal utility of good x is the partial derivative of the utility function with respect to x:

$$MU_x = \frac{\partial}{\partial x}u(x, y) = \frac{\partial}{\partial x}2x^2y = \boxed{4xy}$$

3.B. Find the marginal utility of y.

Solution:

The marginal utility of good y is the partial derivative of the utility function with respect to y:

$$MU_y = \frac{\partial}{\partial y}u(x, y) = \frac{\partial}{\partial y}2x^2y = \boxed{2x^2}$$

3.C. Find the optimal ratio of goods x and y for this consumer.

Solution:

Set the marginal rate of substitution equal to the relative prices:

$$MRS = \frac{P_x}{P_y} \quad \Rightarrow \quad \frac{4xy}{2x^2} = \frac{3}{2} \quad \Rightarrow \quad \frac{2y}{x} = \frac{3}{2} \quad \Rightarrow \quad 4y = 3x \quad \Rightarrow \quad \boxed{y = \frac{3}{4}x}$$

3.D. Find the optimal quantity of good x for this consumer.

Solution:

Insert the optimal ratio found in 3.C. to the budget constraint:

$$3x + 2y = 30 \quad \Rightarrow \quad 3x + 2\left(\frac{3}{4}x\right) = 30 \quad \Rightarrow \quad \frac{9}{2}x = 30 \quad \Rightarrow \quad \boxed{x^* = \frac{20}{3}}$$

Problem 4. The Utility Maximization Problem

Suppose that the consumer is participating in a market consisting of goods x and y. Good x costs \$3, and good y costs \$2 for each unit. The consumer has \$30 as their income, that they will exhaust on consuming goods x and y. The consumer's utility function is given as:

$$u(x, y) = 2x + y$$

4.A. Find the marginal utility of x.

Solution:

The marginal utility of good x is the partial derivative of the utility function with respect to x:

$$MU_x = \frac{\partial}{\partial x} u(x, y) = \frac{\partial}{\partial x}(2x + y) = \boxed{2}$$

4.B. Find the marginal utility of y.

Solution:

The marginal utility of good y is the partial derivative of the utility function with respect to y:

$$MU_y = \frac{\partial}{\partial y} u(x, y) = \frac{\partial}{\partial y}(2x + y) = \boxed{1}$$

4.C. Comparing the per dollar marginal utility of good x and y, determine which good the consumer should purchase.

Solution:

Since x and y are perfect substitutes, the consumer should exclusively purchase the good with the higher per dollar marginal utility:

$$\frac{MU_x}{P_x} = \frac{2}{3} > \frac{1}{2} = \frac{MU_y}{P_y}$$

The consumer should purchase good x exclusively.

4.D. Find the optimal quantity of good x for this consumer.

Solution:

Since the consumer is only purchasing good x:

$$x^* = \frac{M}{P_x} \quad \Rightarrow \quad x^* = \frac{30}{3} \quad \Rightarrow \quad \boxed{x^* = 10}$$

Problem 5. The Utility Maximization Problem

Suppose that the consumer is participating in a market consisting of goods x and y. Good x costs \$3, and good y costs \$2 for each unit. The consumer has \$30 as their income, that they will exhaust on consuming goods x and y. The consumer's utility function is given as:

$$u(x, y) = min\{2x, y\}$$

5.A. Find the optimal ratio of goods x and y for this consumer.

Solution:

The optimal ratio of goods when the goods are perfect complements, we set the arguments of the min function equal to each other:

$$\boxed{2x = y}$$

5.B. Find the optimal quantity of good x for this consumer.

Solution:

Plug in the optimal ratio into the budget constraint to solve for x^*.

$$3x + 2y = 30 \;\Rightarrow\; 3x + 2(2x) = 30 \;\Rightarrow\; 7x = 30 \;\Rightarrow\; \boxed{x^* = \frac{30}{7}}$$