Handout #6: Geometric Sequences

ECON 300: Intermediate Price Theory

Topic 1. Geometric Sequences

A geometric sequence is a sequence that can be defined by the initial value a, and a common ratio r, with n terms. We can express this as:

$$a \quad ar \quad ar^2 \quad ar^3 \quad ar^4 \quad \cdots \quad ar^{n-1}$$

The sum of this sequence can be found using the following formula:

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1} = \sum_{t=1}^{n} ar^{t-1} = a\left(\frac{1-r^n}{1-r}\right)$$

Topic 2. Geometric Series

Suppose that the geometric sequence defined above ends up going infinitely ($n \to \infty$). Then, the sum of this sequence, which is called the geometric series, can be found as:

$$S = a + ar + ar^2 + \cdots$$

Provided that $a \neq 0$, $-1 < r < 1$, and $r \neq 0$, the sum of this sequence can be found using the following formula:

$$S = a + ar + ar^2 + \cdots = \sum_{t=0}^{\infty} ar^t = \frac{a}{1-r}$$

Topic 3. Some Use Cases: Infinite Money?

Suppose you are given an investment opportunity. You must pay an up-front cost now, and then this investment will yield \$100 every single year at the end of each year until the end of time. Meanwhile, we assume that the interest rate remains at 5% over the entire duration. Then the present value of this investment opportunity can be calculated as:

$$PV = 100 \cdot \frac{1}{1+0.05} + 100 \cdot \left(\frac{1}{1+0.05}\right)^2 + 100 \cdot \left(\frac{1}{1+0.05}\right)^3 + \cdots = \frac{\frac{100}{1.05}}{1 - \frac{1}{1.05}} = 2,000$$

Topic 4. Some Use Cases: Bonds?

Suppose you are given another investment opportunity. The setup is the same as the previous case, but this time the payment ends in 5 years. Then, we can find the present value of this investment opportunity as:

$$PV = \frac{100}{1+0.05} + \frac{100}{(1+0.05)^2} + \frac{100}{(1+0.05)^3} + \frac{100}{(1+0.05)^4} + \frac{100}{(1+0.05)^5}$$

Using the formula from **Topic 1**, we can find the present value as:

$$PV = \frac{100}{1.05} \left\{ \frac{1 - \left(\frac{1}{1+0.05}\right)^5}{1 - \frac{1}{1+0.05}} \right\} \simeq 432.95$$