Handout #4: Partial Derivatives

ECON 300: Intermediate Price Theory

Topic 1. Partial Derivatives

Since we will *usually* be dealing with more than two variables, we must cover the concept of partial derivatives. When it comes to the consumers' utility maximization problem, we will typically consider a market with two goods: x and y. When we consider the producers' cost minimization problem, we will consider two inputs: K and L. So, we need to find a way to deal with two variables.

We use partial derivatives to indicate that we are taking the derivative with respect to one of the variables, instead of many. To clearly communicate this, instead of the d we used in the previous case, we use ∂ to indicate that we are taking the derivative over one specific variable.

$$\frac{\partial}{\partial x} f(x,y) \equiv f_x(x,y), \quad \text{and} \quad \frac{\partial}{\partial y} f(x,y) \equiv f_y(x,y)$$

The rule remains *nearly* identical to the rules we introduced in **Topic 2** of the previous handout. One major change is that any variable that is "not changing" is treated as a constant number, not a variable.

Topic 2. Examples

Let's see what this means with an example. Suppose you are asked to solve the following:

$$\frac{\partial}{\partial x}\left(x^2 + y^2\right)$$

We first separate the terms linked by addition or subtraction:

$$\frac{\partial}{\partial x}\left(x^2\right) + \frac{\partial}{\partial x}\left(y^2\right)$$

Identify the variables that are "not changing"and the ones that are "changing."

$$\frac{\partial}{\partial x}\left(x^2\right) + \frac{\partial}{\partial x}\left(y^2\right)$$

Note that the "not changing"variables are treated as constants, so we have:

$$\frac{\partial}{\partial x}\left(x^2 + y^2\right) = 2x + 0$$

Multiplied Terms

Suppose you are asked to solve the following:

$$\frac{\partial}{\partial x}\left(10x^2y\right)$$

The first task you should perform is to identify the variables that are "not changing"and the ones that are "changing."

$$\frac{\partial}{\partial x}\left(10x^2y\right)$$

Pull out the "not changing" parts and apply the power rule to the "changing" parts:

$$10y \cdot \frac{\partial}{\partial x}\left(x^2\right) = 10y \cdot 2x$$

Rearranging the terms, we reach the conclusion:

$$\frac{\partial}{\partial x}\left(10x^2y\right) = 20xy$$

Mixed Example

We will cover one more example. Suppose this time that we are taking the partial derivative with respect to y instead of x:

$$\frac{\partial}{\partial y}\left(5xy^2 - y^3 + x - 10\right)$$

When terms are added / subtracted, it makes solving easier if we first separate the problem into:

$$\frac{\partial}{\partial y}\left(5xy^2\right) + \frac{\partial}{\partial y}\left(-y^3\right) + \frac{\partial}{\partial y}\left(x\right) + \frac{\partial}{\partial y}\left(-10\right)$$

The first task you should perform in this case remains constant: to identify the variables that are "not changing"and the ones that are "changing."

$$\frac{\partial}{\partial y}\left(5xy^2\right) + \frac{\partial}{\partial y}\left(-y^3\right) + \frac{\partial}{\partial y}\left(x\right) + \frac{\partial}{\partial y}\left(-10\right)$$

Note that the "not changing" parts are treated as constants, and solve:

$$5x \cdot 2y + (-3y^2) + 0 + 0$$

Rearranging the terms, we reach the conclusion:

$$\frac{\partial}{\partial y}\left(5xy^2 - y^3 + x - 10\right) = 10xy - 3y^2$$

Topic 3. Practice Problems Please find the values of:

1. $\dfrac{\partial}{\partial x}\,(x+3y)$

2. $\dfrac{\partial}{\partial y}\,(x+3y)$

3. $\dfrac{\partial}{\partial x}\,(x^2y^2)$

4. $\dfrac{\partial}{\partial y}\,(x^2y^2)$

5. $\dfrac{\partial}{\partial x}\,(xy^2-x^2)$

6. $\dfrac{\partial}{\partial y}\,(xy^2-x^2)$