

Handout #2: Slopes

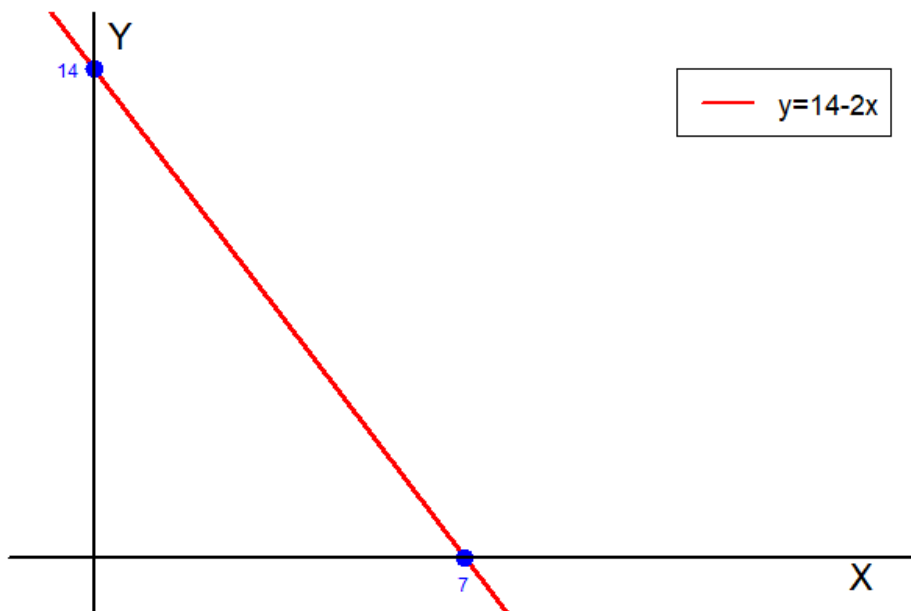
ECON 300: Intermediate Price Theory

Topic 1. Slopes

To determine the slope of any function (graph), you would typically require a minimum of two points and measure the “vertical change” over the “horizontal change.”

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Consider the linear function $f(x) = 14 - 2x$ plotted below:



Consider the two intercepts. When $x = 0$, we know that $f(x) = 14$, and for $f(x) = 0$ to be true, x has to be 7. Thus, the two intercept points are $x = 0, y = 14$ and $x = 7, y = 0$. Therefore, the slope of this function will be:

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{14 - 0}{0 - 7} = -2$$

For these linear functions, the slope remains constant. Therefore, the choice of any two points on the graph is inconsequential, as the result will always be the same. The slope of a linear function can always be calculated using the formula:

$$f(x) = a + bx \Rightarrow \text{Slope} = \begin{cases} -\frac{b}{a} & \text{when } a \neq 0 \\ -b & \text{when } a = 0 \end{cases}$$

As an exercise, please find the slopes of the following linear functions:

1. $f(x) = 32 + 8x$

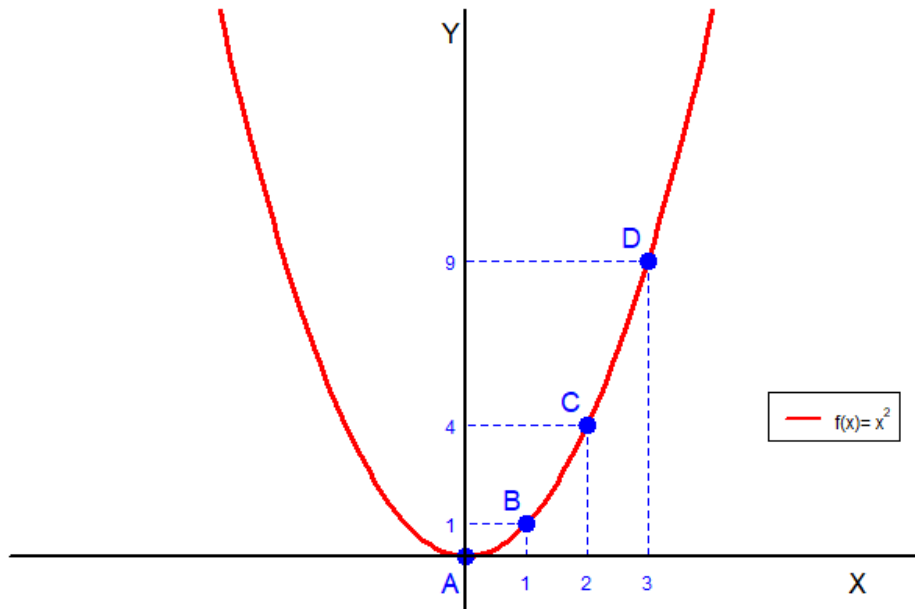
2. $f(x) = -1 + 9x$

3. $f(x) = -3x$

4. $f(x) = 5$

Topic 2. Complications

However, when we encounter functions that are not linear, we begin to encounter issues. Let's consider the quadratic function $f(x) = x^2$. Please note that the figure below is not drawn to scale.



The slope of this function now relies on the chosen points. Let's consider four points: A, B, C, and D, all located on the graph of $f(x) = x^2$. When we calculate the slopes by using point A as the reference and selecting various "end points," we observe that the slope varies between 1 and 3.

$$\bullet \text{ Slope}_{A-B} = \frac{1-0}{1-0} = 1 \quad \bullet \text{ Slope}_{A-C} = \frac{4-0}{2-0} = 2 \quad \bullet \text{ Slope}_{A-D} = \frac{9-0}{3-0} = 3$$

As another exercise, please find the slope when the two chosen points are...

5. C and D