Handout #1: Systems of Equations and Exponents

ECON 300: Intermediate Price Theory

Topic 1. Systems of Equations

Throughout the semester, we will frequently encounter scenarios in which we must solve for a set of unknowns. These unknowns are interconnected through a system of equations. In ECON 300, our focus will primarily be on situations involving two unknowns, interconnected by a set of linear equations. For example, you might be tasked with determining the values of x and y when...

$$2x + 3y = 4 \tag{1}$$
$$x + y = 3 \tag{2}$$

One method of approach is the substitution method. First, we rearrange equation (2):

$$x = 3 - y \tag{3}$$

Then we plug equation (3) into equation (1), and solve for y:

$$2(3 - y) + 3y = 4 \quad \Rightarrow \quad \boxed{y = -2} \tag{4}$$

Then we insert the result from (4) into either equations (1) or (2) to find x:[1]

$$x + (-2) = 3 \quad \Rightarrow \quad \boxed{x = 5}$$

Please complete the exercise by finding the values of x and y when...

1. $\begin{cases} 2x + y = 5 \\ x - 3y = -1 \end{cases}$

[1] In this case, I use equation (2).

Topic 2. Exponents

Another common concept we will encounter throughout the semester pertains to exponents. Let's mention some fundamental facts about exponents...

- $x^a = \overbrace{x \times x \times \cdots \times x}^{a \text{ times}}$
- $x^{-a} = \dfrac{1}{x^a}$
- $x^0 = 1$

The following rules concerning exponents will prove useful as you progress through this course:

- $x^a \times x^b = x^{a+b}$
- $x^a \times y^a = (x \times y)^a$
- $(x^a)^b = x^{a \times b}$
- $\dfrac{x^a}{x^b} = x^{a-b}$
- $\dfrac{x^a}{y^a} = \left(\dfrac{x}{y}\right)^a$

Let's review a few of these rules and examine why they are logical. To illustrate the principles behind these rules, I'll employ $a = 3$ and $b = 2$. Firstly, why does $x^a \times x^b = x^{a+b}$?

$$x^3 \times x^2 = \overbrace{\underbrace{(x \times x \times x)}_{3 \text{ times}} \times \underbrace{(x \times x)}_{2 \text{ times}}}^{3+2 \text{ times}} = x^5$$

Then why is $x^a/x^b = x^{a-b}$.

$$\frac{x^3}{x^2} = \frac{\overbrace{x \times x \times x}^{3 \text{ times}}}{\underbrace{x \times x}_{2 \text{ times}}} = \frac{x \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x}} = x^{3-2} = x$$

How about $x^a \times y^a = (x \times y)^a$?

$$\begin{aligned} x^3 \times y^3 &= \overbrace{(x \times x \times x)}^{3 \text{ times}} \times \overbrace{(y \times y \times y)}^{3 \text{ times}} \\ &= x \times y \times x \times y \times x \times y \\ &= \underbrace{(x \times y) \times (x \times y) \times (x \times y)}_{3 \text{ times}} = (x \times y)^3 \end{aligned}$$

How does $(x^a/y^a) = (x/y)^a$?

$$\frac{x^3}{y^3} = \frac{\overbrace{x \times x \times x}^{3 \text{ times}}}{\underbrace{y \times y \times y}_{3 \text{ times}}} = \underbrace{\left(\frac{x}{y}\right) \times \left(\frac{x}{y}\right) \times \left(\frac{x}{y}\right)}_{3 \text{ times}} = \left(\frac{x}{y}\right)^3$$

Finally why $(x^a)^b = x^{a \times b}$?

$$(x^3)^2 = (\underbrace{x \times x \times x}_{3 \text{ times}})^2 = \underbrace{(x \times x \times x) \times (x \times x \times x)}_{3 \times 2 \text{ times}} = x^6$$

Please complete the following exercises:

2. Simplify: $x^3 \times x^2$

3. Simplify: $\dfrac{x^3}{x}$

4. Simplify: $x^3 \times y^3$

5. Simplify: $6x^2 \times \dfrac{1}{2}x^3$

6. Simplify: $x^{\frac{1}{2}} \times x^3$

7. Simplify: $\dfrac{x^{\frac{1}{2}}}{x^{-\frac{1}{2}}}$

Now for a slightly more challenging exercise: Solve for x and y when...

$$\frac{\frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}}}{\frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}}} = \frac{1}{2}$$
$$4x + 8y = 10$$