Monmouth COLLEGE

- Name: ______________________
- Date: ______________________
- Section: ______________________

ECON 300: Intermediate Price Theory

Problem Set #4: Suggested Solutions

INSTRUCTIONS:

- This problem set is not graded.

Problem 1. Utility Maximization

The consumer is participating in a market with good x and good y. The market prices are given as $P_x = 3$ and $P_y = 2$, respectively. The consumer's income is $M = 45$, and their utility functions are:

$$u(x, y) = min\{2x, 3y\}$$

1.A What does the functional form of the utility function imply about the relationship between goods x and y?

Solution:

The consumer's utility function takes the Leontief form (min function). We use this type of utility function when we know that the goods x and y are **Perfect Complements** that must be used in conjunction with another.

1.B What is the optimal ratio of goods x and y?

Solution:

We find the optimal ratio of consumption by equalizing the arguments of the min function:

$$2x = 3y \quad \Rightarrow \quad \boxed{y = \frac{2}{3}x}$$

1.C What is the mathematical expression of the consumer's budget constraint?

Solution:

Plug in the values of P_x, P_y, and M into the generic form $P_x x + P_y y = M$:

$$\boxed{3x + 2y = 45}$$

1.D Find the optimal bundle (x^*, y^*).

Solution:

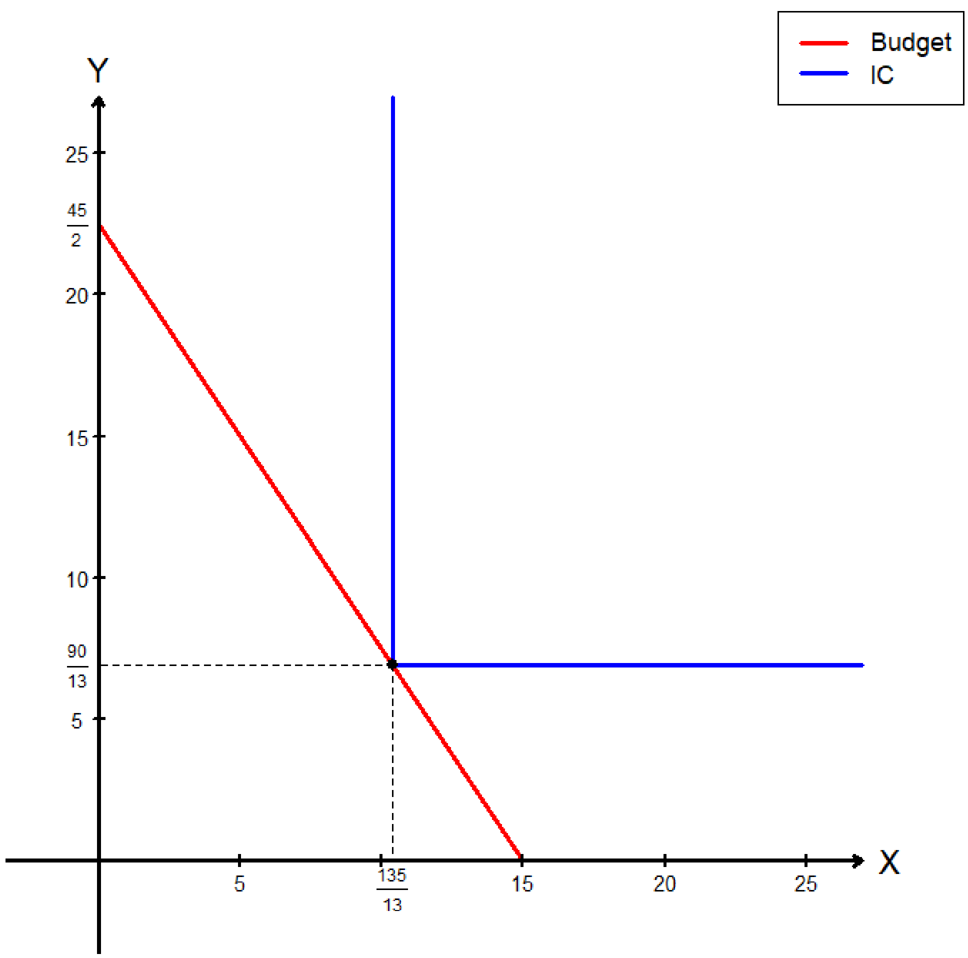
Plug in the optimal ratio we found in 1.B. into the budget constraint we found in 1.C:

$$3x + 2y = 45 \quad \Rightarrow \quad 3x + 2\left(\frac{2}{3}x\right) = 45 \quad \Rightarrow \quad \frac{13}{3}x = 45 \quad \boxed{x^* = \frac{135}{13}}$$

Plug in x^* into the optimal ratio equation from 1.B:

$$y^* = \frac{2}{3}x^* \quad \Rightarrow \quad y^* = \frac{2}{3} \cdot \frac{135}{13} \quad \Rightarrow \quad \boxed{y^* = \frac{90}{13}}$$

1.E Plot and label the consumer's optimization problem in the commodity space.



Problem 2. Utility Maximization

The consumer is participating in a market with good x and good y. The market prices are given as $P_x = 3$ and $P_y = 5$, respectively. The consumer's income is $M = 60$, and their utility functions are:

$$u(x, y) = x^3 y^2$$

2.A Find the expressions for the marginal utilities of good x and good y.

- $MU_x = \frac{\partial}{\partial x} x^3 y^2 = y^2 \cdot 3 \cdot x^{3-1} = \boxed{3x^2y^2}$

- $MU_y = \frac{\partial}{\partial y} x^3 y^2 = x^3 \cdot 2 \cdot y^{2-1} = \boxed{2x^3y}$

2.B What is the optimal ratio of goods x and y?

Solution:

First we find the marginal rate of substitution:

$$MRS = \frac{MU_x}{MU_y} = \frac{3x^2y^2}{2x^3y} = \frac{3y}{2x}$$

Then use the optimality condition by setting the MRS equal to the price ratio:

$$MRS = \frac{P_x}{P_y} \quad \Rightarrow \quad \frac{3y}{2x} = \frac{3}{5} \quad \Rightarrow \quad 15y = 6x \quad \Rightarrow \quad \boxed{y = \frac{2}{5}x}$$

2.C What is the mathematical expression of the consumer's budget constraint?

Solution:

Plug in the values of P_x, P_y, and M into the generic form $P_x x + P_y y = M$:

$$\boxed{3x + 5y = 60}$$

2.D Find the optimal bundle (x^*, y^*).

Solution:

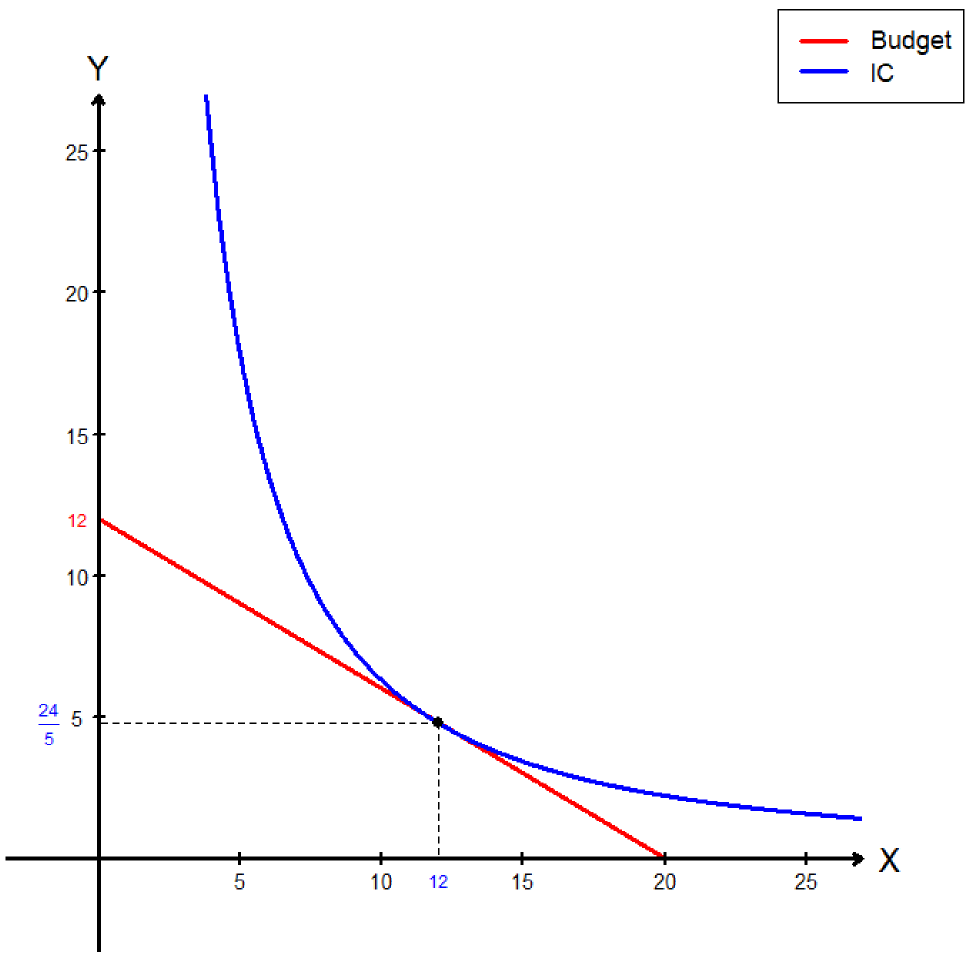
Plug in the optimal ratio we found in 2.B. into the budget constraint we found in 2.C:

$$3x + 5y = 60 \quad \Rightarrow \quad 3x + 5\left(\frac{2}{5}x\right) = 60 \quad \Rightarrow \quad 5x = 60 \quad \boxed{x^* = 12}$$

Plug in x^* into the optimal ratio equation from 2.B:

$$y^* = \frac{2}{5}x^* \quad \Rightarrow \quad y^* = \frac{2}{5} \cdot 12 \quad \Rightarrow \quad \boxed{y^* = \frac{24}{5}}$$

2.E Plot and label the consumer's optimization problem in the commodity space.



Problem 3. Individual to Market Demand

Suppose we have a market for good x with two consumers. Their individual demand functions are:

- Consumer 1: $x_1 = 10 - \frac{1}{4}P_x$
- Consumer 2: $x_2 = 20 - \frac{2}{3}P_x$

3.A Find the inverse demand function for consumer 1.

Solution:

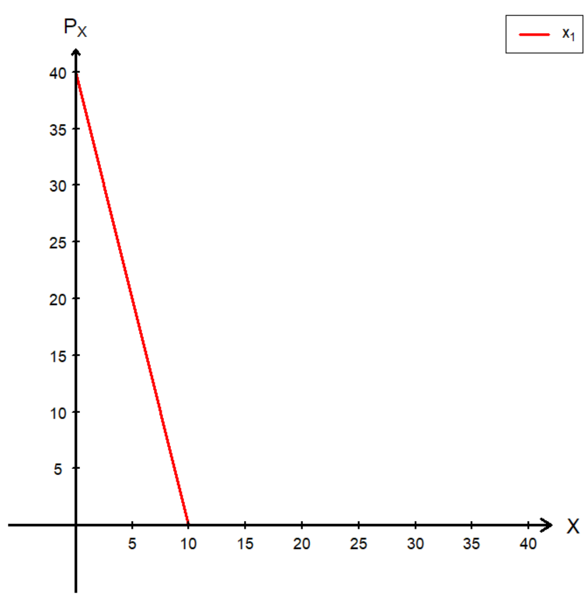
$$x_1 = 10 - \frac{1}{4}P_x \quad \Rightarrow \quad \frac{1}{4}P_x = 10 - x_1 \quad \Rightarrow \quad \boxed{P_x = 40 - 4x_1}$$

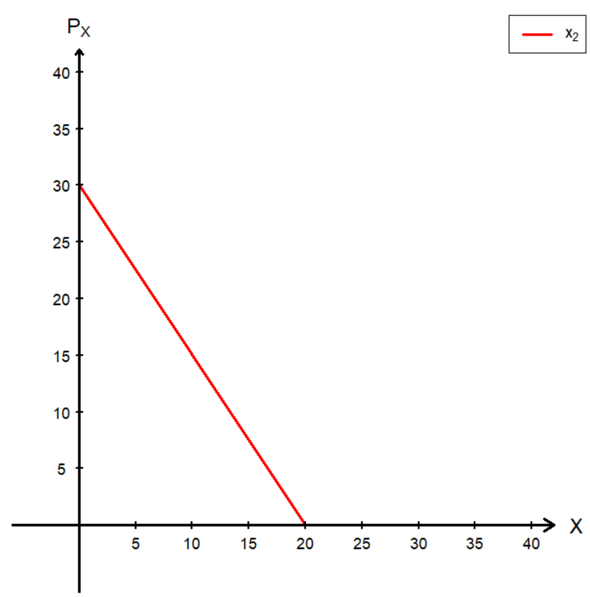
3.B Find the inverse demand function for consumer 2.

Solution:

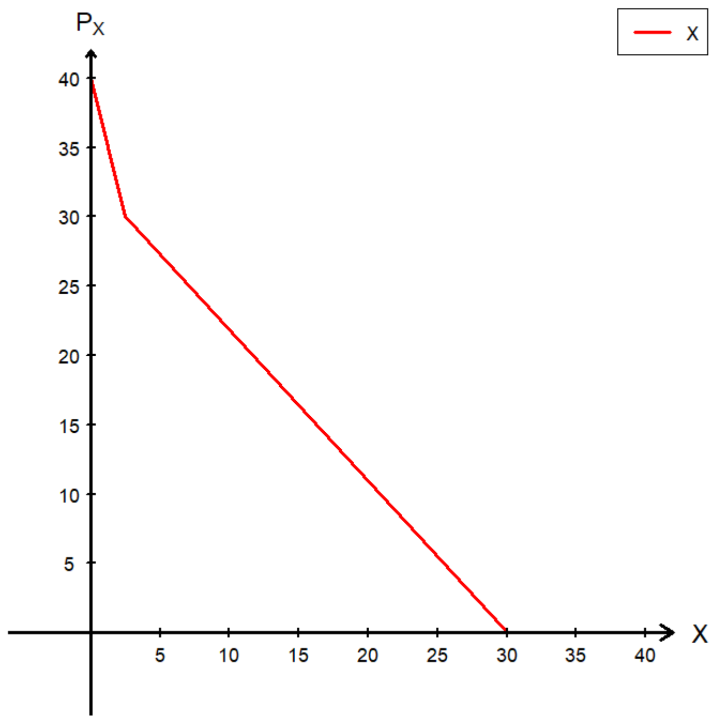
$$x_2 = 20 - \frac{2}{3}P_x \quad \Rightarrow \quad \frac{2}{3}P_x = 20 - x_2 \quad \Rightarrow \quad \boxed{P_x = 30 - \frac{3}{2}x_2}$$

3.C Plot the consumer 1's demand curve to the left, and consumer 2's demand curve to the right.





3.D Plot the market demand for good x.



3.E (ADVANCED) Find the formula for the market demand.

Solution:

The market demand will depend on the market prices. Based on the results from 3.D, we can see that until consumer 2 begins to purchase good x, the market consists of only consumer 1. So, for prices above 30, the market demand is the individual demand of consumer 1.

Meanwhile, when the market price dips below 30, then consumer 2 begins to purchase good x, so the market demand for market prices below 30 should be the (horizontal) sum of the two consumers. So, the inverse market demand should be:

$$\boxed{X(P_x, P_{-x}, M) = \begin{cases} 10 - \frac{1}{4}P_x, & \text{if } 30 < P_x \leq 40 \\ \frac{140}{11} - \frac{2}{11}P_x, & \text{if } 0 \leq P_x \leq 30 \end{cases}}$$

Problem 4. Own Price Elasticities and Revenues

The market price for good x is currently $P_x = 600$, and the market demand for good x is given as:

$$X = 5,000 - 5P_x$$

4.A Calculate the (own) price elasticity of demand for good x.

Solution:

Apply the formula for the (own) price elasticity of demand, and find:

$$\varepsilon_{xp} = \frac{\partial X}{\partial P_x} \cdot \frac{P_x}{X} = -5 \cdot \frac{600}{5000 - 5 \cdot 600} = -5 \cdot \frac{600}{2000} \quad \Rightarrow \quad \boxed{\varepsilon_{xp} = -\frac{3}{2}}$$

4.B Complete the following statement regarding the (own) price elasticity of demand:

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"When the price of good x increases by 1%, then
 the quantity demanded of x decreases by  1.5  %."
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4.C Would you consider the demand for good x to be inelastic? elastic? unit-elastic? Why?

Solution:

Good x is considered to be **elastic**, as the absolute value of price elasticity is greater than 1:

$$|\varepsilon_{xp}| = \frac{3}{2} > 1$$

4.D Calculate the Total Revenue (in terms of price) for the producer of good x.

Solution:

Total revenue is simply price times quantity:

$$TR(P_x) = P_x \cdot X = P_x \cdot (5000 - 5P_x) \quad \Rightarrow \quad \boxed{TR(P_x) = 5000P_x - 5P_x^2}$$

4.E Find the expression for Marginal Revenue (in terms of price) for the producer of good x.

Solution:

Take the partial derivative of TR with respect to P_x:

$$\begin{aligned} MR(P_x) = \frac{\partial TR(P_x)}{\partial P_x} &= \frac{\partial}{\partial P_x}(5000P_x - 5P_x^2) \\ &= \frac{\partial}{\partial P_x}5000P_x - \frac{\partial}{\partial P_x}5P_x^2 \\ &= 5000 - 10P_x \\ &\Rightarrow \boxed{MR(P_x) = 5000 - 10P_x} \end{aligned}$$

4.F Is the producer maximizing their revenue if the market price is set at $P_x = 600$? Why?

Solution:

From the results in 4.E., when the market price is set at $P_x = 600$, marginal revenue is...

$$MR(P_x) = 5000 - 10P_x \quad \Rightarrow \quad MR(P_x) = -1000 < 0$$

This implies that when the producer lowers prices, the total revenue they obtain will increase. Therefore, when the market price is set to be $P_x = 600$, the producer is not maximizing their revenue.

4.G If the price of good x decreases to $P_x = 500$, is the demand for good x inelastic? elastic? unit-elastic? Why?

Solution:

Apply the formula for the (own) price elasticity of demand, and find:

$$\varepsilon_{xp} = \frac{\partial X}{\partial P_x} \cdot \frac{P_x}{X} = -5 \cdot \frac{500}{5000 - 5 \cdot 500} = -5 \cdot \frac{500}{2500} \quad \Rightarrow \quad \boxed{\varepsilon_{xp} = -1}$$

When the market price of good x is set to be 500, the (own) price elasticity of demand is -1, and the demand is unit elastic.

4.H Is the producer maxmizing revenue when the market price is set at $P_x = 500$? Why?

Solution:

Recall from the answer in part 4.E, we have the expression for the marginal revenue:

$$MR(P_x) = 5000 - 10P_x$$

When the market price of good x is set to be 500, the marginal revenue will be 0. This implies that the total revenue of the producer is maximized.

Problem 5. Other Types of Elasticities

The current price of good x is $P_x = 10$, the price of good y is $P_y = 20$, and the overall income level of the economy is $M = 100$. The market demand for good x is given as follows:

$$X = 500 - 30P_x + 5M + 10P_y$$

5.A Calculate the (own) price elasticity of demand for good x.

Solution:

Apply the formula for the own price elasticity:

$$\varepsilon_{xp} = \frac{\partial X}{\partial P_x} \cdot \frac{P_x}{X} = -30 \cdot \frac{10}{500 - 30 \cdot 10 + 5 \cdot 100 + 10 \cdot 20} = -30 \cdot \frac{10}{900} = -\frac{1}{3}$$

5.B Complete the following statement regarding the (own) price elasticity of demand:

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"When the price of good x increases by 1%, then
 the quantity demanded of x decreases by  0.33 %."
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5.C Would you consider the demand for good x to be elastic? Why?

Solution:

Since the absolute value of price elasticity is less than 1, the demand is **inelastic**.

5.D Is good x an ordinary good or a Giffen good? Why?

Solution:

Since the price elasticity is negative, it means that for a positive change in price ($\Delta P_x \uparrow$), the change quantity demanded is negative ($\Delta x \downarrow$). This means that the good is an **ordinary good**.

5.E If the price of good x increases to $P_x = 30$, is the demand for good x elastic? Why?

Solution:

Apply the formula for the own price elasticity:

$$\varepsilon_{xp} = \frac{\partial X}{\partial P_x} \cdot \frac{P_x}{X} = -30 \cdot \frac{30}{500 - 30 \cdot 30 + 5 \cdot 100 + 10 \cdot 20} = -30 \cdot \frac{30}{300} = -3$$

Since the absolute value of price elasticity is greater than 1, the demand is **elastic**.

5.F Calculate the income elasticity of demand when $P_x = 30$, $P_y = 20$, $M = 100$.

Solution:

Apply the formula for income elasticity:

$$\varepsilon_{xm} = \frac{\partial X}{\partial M} \cdot \frac{M}{X} = 5 \cdot \frac{100}{500 - 30 \cdot 30 + 5 \cdot 100 + 10 \cdot 20} = 5 \cdot \frac{100}{300} = \frac{5}{3}$$

5.G Complete the following statement regarding the income elasticity of demand:

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"When the consumers' income increases by 1%, then
 the quantity demanded of x increases by  1.67 %."
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5.H Is good x a luxury good or a necessary good? Why?

Solution:

Since the quantity demanded increases at a greater rate compared to the increase in income, good x is a **luxury good**.

5.I Calculate the cross price elasticity of demand when $P_x = 30, P_y = 20, M = 100$.

Solution:

Apply the formula for income elasticity:

$$\varepsilon_{xy} = \frac{\partial X}{\partial P_y} \cdot \frac{P_y}{X} = 10 \cdot \frac{20}{500 - 30 \cdot 30 + 5 \cdot 100 + 10 \cdot 20} = 10 \cdot \frac{20}{300} = \frac{2}{3}$$

5.J Complete the following statement regarding the income elasticity of demand:

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"When the price of good y increases by 1%, then
 the quantity demanded of x increases by  0.67 %."
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5.K Is good y a complement to good x? Is it a substitute to good x? Why?

Solution:

When the price of good y increases ($\Delta P_y \uparrow$), the quantity demanded of x increases ($\Delta x \uparrow$). This happens when the two goods x and y are **substitutes**.