CONCORDIA COLLEGE
MOORHEAD, MINNESOTA

- Name: _____
- Date: _____
- Section: ○ Mon/Wed ○ Tue/Thu

BUSN 320: Business Statistics

Problem Set #3: Suggested Solutions

Fall 2026

INSTRUCTIONS:

- These practice problems are intended to help you prepare for Quiz #3.
- You may use your textbook, class notes, calculator, and other course resources while completing these problems.
- The problem set consists of three parts: Definitions, True / False, and Short Answers.
- The practice problems are broader than the quiz and do not necessarily represent the exact questions that will appear on Quiz #3.

Problem 1. Definitions

The following terms are fair game for definition questions on Quiz #3. You should be able to define each term and distinguish it from closely related concepts.

Chapter 7

- Finite Population
- Infinite Population
- Sampling with Replacement
- Sampling without Replacement
- Estimand
- Estimator
- Estimate
- Point Estimator
- Population Proportion
- Sample Proportion
- Sampling Distribution
- Standard Error
- Central Limit Theorem
- Unbiasedness
- Efficiency
- Consistency
- Sampling Error
- Nonsampling Error
- Coverage Error
- Nonresponse Error
- Measurement Error

Chapter 8

- Interval Estimate
- Confidence Interval
- Confidence Level
- Margin of Error
- Critical Value
- t-Distribution

Problem 2. True / False

For each statement, indicate whether it is `TRUE` or `FALSE`. A correct `TRUE` response will receive full credit. For a `FALSE` statement, you must briefly explain why the statement is false to receive full credit.

2.A. Two random samples from the same population can produce different estimates even when the same estimator is used.

- `TRUE`

2.B. The standard error of the sample mean measures how much individual observations vary around their sample mean.

- `FALSE`
  Standard error measures variation in the sample mean across random samples, rather than variation among individual observations.

2.C. If the population distribution is normal, the sampling distribution of the sample mean is normal even when the sample size is small.

- `TRUE`

2.D. The central limit theorem guarantees that the sampling distribution of the sample mean is approximately normal whenever $n \geq 30$, regardless of the population's shape.

- `FALSE`
  $n \geq 30$ is a guideline, not a guarantee. Strongly skewed populations may require larger samples.

2.E. Two estimators can both be unbiased even though one produces estimates that vary more across random samples.

- `TRUE`

2.F. An estimator that is biased for a given sample size cannot be consistent.

- `FALSE`
  A biased estimator can be consistent if it becomes increasingly concentrated around the population parameter as sample size increases.

Problem 2. True / False (continued)

For each statement, indicate whether it is TRUE or FALSE. A correct TRUE response will receive full credit. For a FALSE statement, you must briefly explain why the statement is false to receive full credit.

2.G. If $p = 0.10$ and $n = 40$, the sampling distribution of the sample proportion satisfies the conditions for a normal approximation used in this course.

- FALSE
  $np = 40 \cdot 0.1 = 4 < 5$, so the normal-approximation conditions used in this course are not satisfied.

2.H. If a sample produces a 95% confidence interval of $(40, 50)$ for a population mean, the sample mean is 45 and the margin of error is 5.

- TRUE

2.I. Across repeated random samples, approximately 95% of the intervals constructed using a valid 95% confidence interval procedure will contain the population parameter.

- TRUE

2.J. Using the same sample, a 90% confidence interval for a population mean is wider than a 99% confidence interval constructed using the same method.

- FALSE
  Using the same sample and method, a 90% confidence interval is narrower than a 99% interval.

2.K. Holding the confidence level and population standard deviation fixed, reducing the desired margin of error by half requires approximately twice as many observations.

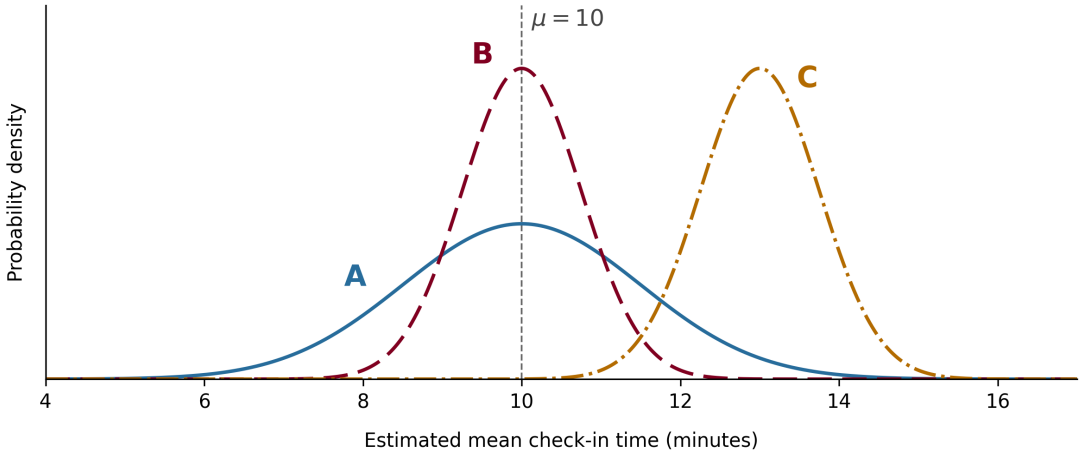
- FALSE
  Halving the margin of error requires approximately four times as many observations, holding confidence level and population standard deviation fixed.

2.L. Surveying more customers from the same incomplete sampling frame will necessarily eliminate coverage error.

- FALSE
  Increasing sample size does not correct the exclusion of relevant groups from the sampling frame.

Problem 3. Short Answers

3.A. A hotel is comparing three estimators, A, B, and C, of the population mean check-in time. Each estimator uses a random sample of 50 check-ins. For this comparison, suppose the true population mean is known to be $\mu = 10$ minutes. The graph shows the theoretical sampling distribution of each estimator across repeated random samples of the same size.



- What does a value on the horizontal axis represent? Does each curve describe individual guests' check-in times or estimates calculated from repeated samples?
  - Each horizontal-axis value is a possible estimate of the population mean check-in time, calculated from a sample of 50 check-ins. Each curve represents the estimator's sampling distribution across repeated samples, rather than the distribution of individual check-in times.
- Which estimators are unbiased? For any biased estimator, indicate whether it tends to overestimate or underestimate the population mean. Explain using the graph.
  - A and B are unbiased because their sampling distributions are centered at the population mean $\mu = 10$. C is biased upward because its sampling distribution is centered above 10 minutes, so it systematically overestimates the population mean.
- Among the unbiased estimators, which is more efficient? Explain using the standard errors of their sampling distributions.
  - B is more efficient than A. Both are unbiased, but B has a smaller standard error, shown by its narrower sampling distribution. Its estimates are more concentrated around the population mean.
- An analyst argues that estimator C is preferable to estimator A because its sampling distribution is narrower. What does this argument overlook?
  - The argument overlooks bias. Although C varies less across samples than A, it is concentrated around the wrong value. A narrower sampling distribution alone does not establish that an estimator is preferable.

Problem 3. Short Answers (continued)

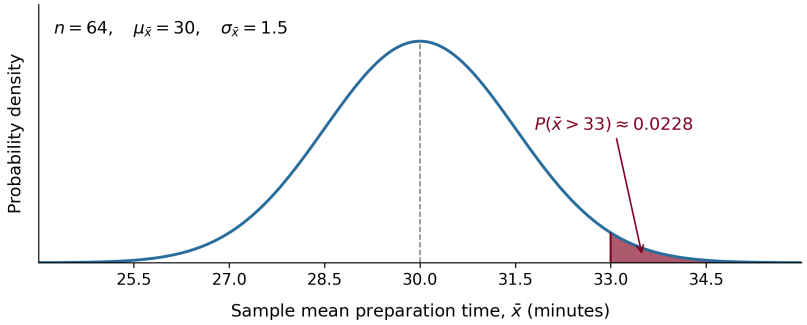
3.B. The time required to prepare a hotel room for an arriving guest has a population mean of 30 minutes and a population standard deviation of 12 minutes. The population distribution is moderately right-skewed. A random sample of 64 room preparations is selected. Assume that the selected preparation times are independent and come from the same population.

- Find the expected value and standard error of the sample mean preparation time.

$$E(\bar{\mathrm{x}}) = \mu = 30 \qquad \text{and} \qquad \sigma_{\bar{\mathrm{x}}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{64}} = 1.5$$

- Explain why the sampling distribution of the sample mean can be approximated by a normal distribution even though individual preparation times are right-skewed.
  - By the central limit theorem, the sampling distribution of the sample mean is approximately normal for a sufficiently large random sample. Here, $n = 64$ supports this approximation for the moderately right-skewed population, given the stated independence assumption.

- Sketch the approximate sampling distribution of the sample mean. Label its mean and the value 33 minutes on the horizontal axis. Shade the region representing the probability that the sample mean exceeds 33 minutes.

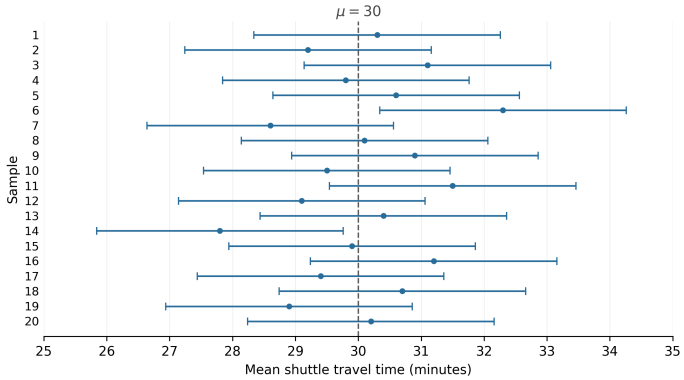


- Calculate the probability that the mean preparation time for the 64 rooms exceeds 33 minutes. Show your standardization and Z-table lookup.

$$z = \frac{33 - 30}{1.5} = 2 \quad \Rightarrow \quad P(\bar{\mathrm{x}} > 33) \simeq P(Z > 2) = 1 - 0.9772 = 0.0228$$

Problem 3. Short Answers (continued)

3.C. An airport shuttle company studies a population of trips with a mean travel time of 30 minutes and a known standard deviation of 8 minutes. The graph illustrates 95% confidence intervals from 20 independent random samples, each containing 64 trips. Assume that the conditions for the usual z-confidence interval hold. Each dot represents a sample mean, and each horizontal segment represents its confidence interval. The dashed vertical line marks the population mean.



- Identify the samples whose intervals do not contain the population mean. What percentage of the 20 intervals contain it?
  - The intervals for samples 6 and 14 do not contain $\mu = 30$. So, $18/20 \times 100 = 90\%$.

- Does the percentage in this graph contradict the use of a 95% confidence interval procedure? Explain what the 95% confidence level means across repeated random samples.
  - The observed 90% coverage does not contradict a 95% confidence interval procedure. Across many repeated random samples, approximately 95% of the intervals constructed using this procedure will contain the population mean. The percentage containing μ can vary from one batch to another. It need not equal exactly 95% in each batch.

- Explain why the intervals have different centers but the same width. If each interval were reconstructed at a 99% confidence level using the same sample, what would happen to its center and width?
  - The intervals have different centers because their sample means vary across random samples. They have the same width because they use the same confidence level, population standard deviation, and sample size.
  - Reconstructing each interval at 99% confidence leaves its center unchanged but increases its margin of error.

$$E_{95\%} = 1.96 \cdot \frac{8}{\sqrt{64}} = 1.96 \qquad E_{99\%} = 2.576 \cdot \frac{8}{\sqrt{64}} = 2.576$$

Problem 3. Short Answers (continued)

3.D. A travel agency wants to estimate population mean spending for two different services. In each case, assume that the observations form an independent random sample from the population described.

Case 1. A sample of 64 airport transfers has a sample mean cost of \$42. The population standard deviation is known to be \$8. Assume that the sampling distribution of the sample mean is approximately normal.

Case 2. A sample of 16 customers has a sample mean spending of \$240 on optional vacation activities and a sample standard deviation of \$60. The population standard deviation is unknown. Assume that spending in this population is normally distributed.

- For each case, identify whether to use a standard normal distribution or a t-distribution to construct a confidence interval for the population mean. Explain your choices and identify the degrees of freedom where applicable.
  - Case 1: Use the standard normal distribution, because σ is known, and the sampling distribution of the sample mean is approximately normal.
  - Case 2: Use the t-distribution with degrees of freedom 15, because σ is unknown and is estimated using $\mathtt{s}$. The population is normal, supporting its use with a small sample.

- Construct a 95% confidence interval for the population mean airport transfer cost in Case 1. Show the standard error, critical value, and margin of error.
  - The standard error is $SE = \frac{\sigma}{\sqrt{n}} = \frac{8}{8} = 1$.
  - For 95% confidence, $z_{0.025} = 1.96$, so $E = z_{0.025} \cdot SE = 1.96 \cdot 1 = 1.96$.
  - The confidence interval is, $\bar{\mathtt{x}} \pm E = 42 \pm 1.96 \simeq (\$40.04, \$43.96)$.

- Construct a 90% confidence interval for the population mean spending on optional activities in Case 2. Show the estimated standard error, critical value, and margin of error.
  - The standard error is estimated as $\widehat{SE} = \frac{\mathtt{s}}{\sqrt{n}} = \frac{60}{4} = 15$.
  - For 90% confidence, $t_{0.05,15} = 1.753$, so $E = t_{0.05,15} \cdot \widehat{SE} = 1.753 \cdot 15 = 26.295$.
  - The confidence interval is, $\bar{\mathtt{x}} \pm E = 240 \pm 26.295 \simeq (\$213.71, \$266.30)$.

- Interpret your confidence interval for Case 2 in context. Would it be appropriate to conclude that 90% of individual customers spend an amount within this interval? Explain.
  - We are 90% confident that the population mean spending on optional vacation activities is between approximately \$213.71 and \$266.30.
  - It would be inappropriate to conclude that 90% of individual customers spend within this interval. The interval estimates the population mean, rather than the range containing a specified percentage of individual spending amounts.

Problem 3. Short Answers (continued)

3.E. A hotel chain wants to estimate the proportion of all its guests during the current month who are satisfied with its breakfast service. It selects a random sample of 200 guests from that population. Of those sampled, 160 report being satisfied. Assume that responses are independent and that every selected guest responds.

- Calculate the sample proportion of satisfied guests and its estimated standard error.

$$\bar{\mathrm{p}} = \frac{160}{200} = 0.8 \qquad \widehat{SE} = \sqrt{\frac{\bar{\mathrm{p}} \cdot (1 - \bar{\mathrm{p}})}{n}} = \sqrt{\frac{0.8 \cdot 0.2}{200}} \simeq 0.0283$$

- Check whether the sample satisfies the conditions for constructing an approximate normal confidence interval for the population proportion.

$$n \cdot \bar{\mathrm{p}} = 160 \geq 5 \qquad \text{and} \qquad n \cdot (1 - \bar{\mathrm{p}}) = 40 \geq 5$$

  – Both conditions are satisfied. Together with the stated random sampling and independence assumptions, these support the approximate normal confidence interval.

- Construct and interpret a 95% confidence interval for the population proportion of guests satisfied with breakfast service. Show the critical value and margin of error.

$$E = z_{0.025} \cdot \widehat{SE} = 1.96 \cdot \sqrt{\frac{0.8 \cdot 0.2}{200}} \simeq 0.0554$$

  – So, the confidence interval is $\bar{\mathrm{p}} \pm E \simeq 0.8 \pm 0.0554 = (0.7446, 0.8554)$.
  – We are 95% confident that between approximately 74.46% and 85.54% of all guests during the current month are satisfied with breakfast service.

- Suppose the hotel had instead sampled only loyalty-program members and obtained the same sample proportion. Would the resulting interval necessarily provide a reliable estimate for all guests? Identify the potential error and explain whether surveying more loyalty-program members would resolve it.

  – The interval would not necessarily provide a reliable estimate for all guests. Excluding guests who are not loyalty-program members creates coverage error, and their satisfaction may differ from that of members.
  – Surveying more loyalty-program members can reduce sampling variability and narrow the interval, but it does not correct the exclusion of nonmembers.

Problem 3. Short Answers (continued)

3.F. A tourism office is planning two separate studies using independent random samples. The first will estimate the population mean daily spending of visitors. A reliable planning value for the population standard deviation is \$40. The second will estimate the population proportion of visitors who use public transportation. Both studies will use a 95% confidence level. Round each required sample size up to the nearest whole number.

- Determine the sample size needed to estimate population mean daily spending with a margin of error of no more than \$10.
  - With 95% confidence, $z_{0.025} = 1.96$. We then use the population standard deviation $\sigma = 40$ and desired margin of error $E = 10$:
$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{1.96 \cdot 40}{10}\right)^2 = 61.4656$$
  - Round up to obtain $n = 62$ visitors.

- For the public transportation study, no reliable prior estimate of the population proportion is available. Identify the planning value you should use and explain why it is appropriate.
  - Use $\mathrm{p}^* = 0.5$ because no reliable prior estimate is available. This value maximizes $\mathrm{p}^* \cdot (1 - \mathrm{p}^*)$, producing the largest required sample size for the specified confidence level and margin of error.

- Determine the sample size needed to estimate the population proportion using public transportation with a margin of error of no more than 5 percentage points.
  - A margin of error of 5 percentage points means $E = 0.05$:
$$n = \frac{z_{\alpha/2}^2 \cdot \mathrm{p}^* \cdot (1 - \mathrm{p}^*)}{E^2} = \frac{1.96^2 \cdot 0.5 \cdot 0.5}{0.05^2} = 384.16$$
  - Round up to obtain $n = 385$ visitors.

- If the desired margin of error in each study were cut in half, by what factor would the required sample size increase before rounding? Explain using the sample-size formulas.
  - Both sample size formulas have E^2 in the denominator. Replacing E with $E/2$ therefore gives:
$$n_{new} = \frac{1}{\left(\frac{1}{2}\right)^2} \cdot n_{original} = 4 \cdot n_{original}$$
  - Thus, halving the desired margin of error requires four times the sample size before rounding, holding the other planning values fixed.