CONCORDIA COLLEGE
MOORHEAD, MINNESOTA

- Name: _____
- Date: _____
- Section: ○ Mon/Wed ○ Tue/Thu

BUSN 320: Business Statistics

Problem Set #2: Suggested Solutions

Fall 2026

INSTRUCTIONS:

- These practice problems are intended to help you prepare for Quiz #2.
- You may use your textbook, class notes, calculator, and other course resources while completing these problems.
- The problem set consists of three parts: Definitions, True / False, and Short Answers.
- The practice problems are broader than the quiz and do not necessarily represent the exact questions that will appear on Quiz #2.

Problem 1. Definitions

The following terms are fair game for definition questions on Quiz #2. You should be able to define each term and distinguish it from closely related concepts.

Chapter 4

- Probability
- Random Experiment
- Sample Space
- Sample Point
- Event
- Classical Method
- Relative Frequency Method
- Subjective Method
- Conditional Probability

Chapter 5

- Random Variable
- Discrete Random Variable
- Continuous Random Variable
- Probability Distribution
- Empirical Discrete Distribution
- Probability Mass Function (PMF)
- Expected Value
- Variance
- Standard Deviation
- Bivariate Probability Distribution
- Marginal Probability
- Discrete Uniform Probability Distribution
- Bernoulli Trial
- Binomial Probability Distribution
- Poisson Probability Distribution

Chapter 6

- Probability Density Function (PDF)
- Z-table
- Normal Probability Distribution
- Standard Normal Probability Distribution
- Continuous Uniform Probability Distribution

Problem 2. True / False

For each statement, indicate whether it is TRUE or FALSE. A correct TRUE response will receive full credit. For a FALSE statement, you must briefly explain why the statement is false to receive full credit.

2.A. An event may contain every sample point in the sample space.

- TRUE

2.B. The classical method assigns equal probabilities to all possible outcomes, even when some outcomes are more likely than others.

- FALSE
  The classical method applies when all possible outcomes are equally likely. It is not appropriate when some outcomes are more likely than others.

2.C. If $P(A) = 0.40$ and $P(B) = 0.30$, then $P(A \cup B)$ must equal 0.70.

- FALSE
  The addition rule gives
  $$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$
  The probability equals 0.70 only if $P(A \cap B) = 0$. Otherwise, we must subtract the probability of the overlap.

2.D. When calculating $P(A \mid B)$, we restrict the sample space to the outcomes belonging to event B.

- TRUE

2.E. A discrete random variable must have a finite number of possible values.

- FALSE
  A discrete random variable may have either a finite number or a countably infinite number of possible values.

2.F. A discrete probability distribution can be valid even if one of its possible values has a probability of zero.

- TRUE

2.G. The expected value of a random variable must be one of the values that the random variable can actually take.

- FALSE
  The expected value is a probability-weighted average of the possible values. This average does not have to be a value that the random variable can actually take.

Problem 2. True / False (continued)

For each statement, indicate whether it is TRUE or FALSE. A correct TRUE response will receive full credit. For a FALSE statement, you must briefly explain why the statement is false to receive full credit.

2.H. If a random variable is measured in dollars, its variance is measured in squared dollars and its standard deviation is measured in dollars.

- TRUE

2.I. A marginal probability describes the probability of a value of one random variable, regardless of the value of the other random variable.

- TRUE

2.J. A binomial distribution can describe the number of successes in a fixed number of independent Bernoulli trials even when the probability of success differs across trials.

- FALSE
  A binomial distribution requires the probability of success to be the same for every trial.

2.K. For a continuous random variable, the height of the probability density function at x equals $P(X = x)$.

- FALSE
  The height of the probability density function represents density, rather than probability. Probabilities are calculated as areas under the density function over intervals. For a continuous random variable, $P(X = x) = 0$.

2.L. If X has a continuous uniform distribution between 0 and 10, then $P(1 \leq X \leq 3) = P(6 \leq X \leq 8)$.

- TRUE

2.M. Increasing the standard deviation of a normal distribution while keeping its mean fixed makes the curve narrower and taller.

- FALSE
  Increasing the standard deviation makes the curve wider and flatter. Its center remains unchanged because the mean is fixed.

2.N. If X follows a normal distribution with mean 100 and standard deviation 10, then $P(X \leq 110) = P(Z \leq 1)$, where Z follows the standard normal distribution.

- TRUE

Problem 3. Short Answers

3.A. A hotel records the following information about 200 reservations. Let D be the event that a reservation was booked directly, and C be the event that it was canceled. Select one reservation at random.

	Canceled	Not Canceled	Total
Booked Directly	12	68	80
Booked Through a Third Party	36	84	120
Total	48	152	200

- Find $P(D)$ and $P(C)$.

$$P(D) = \frac{80}{200} = \boxed{0.40} \qquad P(C) = \frac{48}{200} = \boxed{0.24}$$

- Find $P(D \cap C)$. Explain what this probability represents.

$$P(D \cap C) = \frac{12}{200} = \boxed{0.06}$$

  - There is a 6% probability that a randomly selected reservation was both booked directly and canceled.

- Find $P(D \cup C)$.

$$P(D \cup C) = P(D) + P(C) - P(D \cap C) = 0.40 + 0.24 - 0.06 = \boxed{0.58}$$

- Find $P(C \mid D)$ and $P(C \mid D^c)$.

$$P(C \mid D) = \frac{12}{80} = \boxed{0.15} \qquad P(C \mid D^c) = \frac{36}{120} = \boxed{0.30}$$

- Based on your answers, which booking method has a higher cancellation rate? Explain.
  - Third-party bookings have a higher cancellation rate. Among reservations booked through a third party, 30% were canceled, compared with 15% of reservations booked directly.

Problem 3. Short Answers (continued)

3.B. A café estimates the following probability distribution for the number of unsold pastries at closing, X.

x	0	1	2	3	4
$P(X = x)$	0.10	0.20	0.40	0.20	0.10

- Explain why this is a valid probability mass function.

$$0.1 + 0.2 + 0.4 + 0.2 + 0.1 = \boxed{1}$$

  - Every probability is nonnegative, and the probabilities sum to 1.

- Find the probability that at least three pastries remain unsold.

$$P(X \geq 3) = P(X = 3) + P(X = 4) = 0.2 + 0.1 = \boxed{0.30}$$

- Calculate the expected value of X and interpret it.

$$E(X) = \sum_x xP(x) = 0(0.1) + 1(0.2) + 2(0.4) + 3(0.2) + 4(0.1) = \boxed{2}$$

  - Over many days, the café would have an average of approximately two unsold pastries per day.

- Calculate the variance and standard deviation of X.

$$\begin{aligned} Var(X) &= \sum_x \{x - E(X)\}^2 P(x) \\ &= \sum_x (x-2)^2 P(x) = 4(0.1) + 1(0.2) + 0(0.4) + 1(0.2) + 4(0.1) = \boxed{1.2} \\ \sigma &= \sqrt{Var(X)} = \sqrt{1.20} \simeq \boxed{1.095} \end{aligned}$$

- Each unsold pastry represents \$3 in lost revenue. Find the expected lost revenue per day.

$$3 \cdot E(X) = 3(2) = \boxed{\$6}$$

Problem 3. Short Answers (continued)

3.C. Let X be the number of canceled morning flights and Y the number of canceled evening flights on a particular route during a randomly selected day. Their joint probability distribution is given below.

	$Y=0$	$Y=1$	$Y=2$
$X=0$	0.30	0.10	0.05
$X=1$	0.10	0.20	0.05
$X=2$	0.05	0.05	0.10

- Find the marginal probability distributions of X and Y.
  - Add across rows to obtain the distribution of X, and down columns to obtain the distribution of Y.

x or y	0	1	2
$P(X=x)$	0.45	0.35	0.20
$P(Y=y)$	0.45	0.35	0.20

- Find the probability that there are no canceled flights in either the morning or the evening.

$$P(X=0, Y=0) = \boxed{0.30}$$

- Let $W = X + Y$. Construct the probability distribution of W.
  - Add the joint probabilities for all combinations with the same total.

w	Possible (X, Y) values	$P(W=w)$
0	$(0,0)$	0.30
1	$(0,1), (1,0)$	0.20
2	$(0,2), (1,1), (2,0)$	0.30
3	$(1,2), (2,1)$	0.10
4	$(2,2)$	0.10

Problem 3. Short Answers (continued)

3.C. (continued).

	$Y=0$	$Y=1$	$Y=2$
$X=0$	0.30	0.10	0.05
$X=1$	0.10	0.20	0.05
$X=2$	0.05	0.05	0.10

- Calculate $E(X)$, $E(Y)$, and $E(W)$. What relationship do you observe? Recall that $W = X + Y$.

$$E(X) = 0(0.45) + 1(0.35) + 2(0.20) = \boxed{0.75}$$
$$E(Y) = 0(0.45) + 1(0.35) + 2(0.20) = \boxed{0.75}$$
$$E(W) = 0(0.30) + 1(0.20) + 2(0.30) + 3(0.10) + 4(0.10) = \boxed{1.50}$$

  - The expected total equals the sum of the two expected values.

$$E(W) = E(X) + E(Y) = 0.75 + 0.75 = 1.50$$

- Given that $\text{Var}(X) = \text{Var}(Y) = 0.5875$ and $\text{Var}(W) = 1.775$, calculate the covariance between X and Y. Interpret its sign.

$$\text{Cov}(X,Y) = \frac{\text{Var}(W) - \text{Var}(X) - \text{Var}(Y)}{2} = \frac{1.775 - 0.5875 - 0.5875}{2} = \boxed{0.30}$$

  - The positive covariance indicates that morning and evening cancellations tend to move together. Days with more morning cancellations tend to have more evening cancellations.

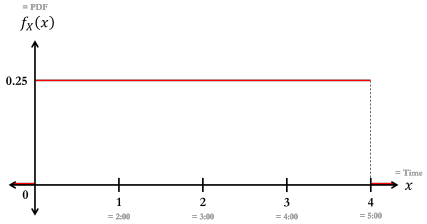
Problem 3. Short Answers (continued)

3.D. A delivery company gives a customer a delivery window from 1:00 PM to 5:00 PM. Assume the arrival time follows a continuous uniform distribution. Let X be the number of hours after 1:00 PM until the delivery arrives.

- State the distribution of X and determine the height of its probability density function.

$$X \sim U(0,4) \qquad \text{Height} = \frac{1}{4-0} = \boxed{0.25}$$

- Sketch the probability density function and label its axes.

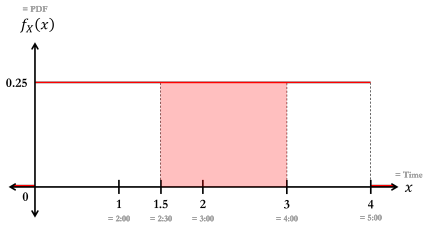


- Find the probability that the delivery arrives before 2:00 PM.

$$P(X < 1) = (1-0)(0.25) = \boxed{0.25}$$

- Find the probability that the delivery arrives between 2:30 PM and 4:00 PM. Shade the corresponding area on your sketch.

$$P(1.5 \leq X \leq 3) = (3-1.5)(0.25) = \boxed{0.375}$$



- Find the probability that the delivery arrives at exactly 3:00 PM. Explain why your answer does not mean that arrival at 3:00 PM is impossible.

$$P(X = 2) = 0(0.25) = \boxed{0}$$

  - A single point has zero width and therefore zero area under the PDF. Arrival at 3:00 PM is a possible value, but a continuous probability model assigns probability zero to each individual value.

Problem 3. Short Answers (continued)

3.E. The time required to process a customer's application is normally distributed with a mean of 40 minutes and a standard deviation of 8 minutes. Use the Z-table and show your standardization steps.

- Find the probability that an application takes less than 32 minutes.

$$P(X < 32) = P(Z < -1) \approx \boxed{0.1587} \qquad \because z = \frac{32 - 40}{8} = -1$$

- Find the probability that an application takes more than 52 minutes.

$$P(X > 52) = 1 - P(Z \leq 1.50) \simeq 1 - 0.9332 = \boxed{0.0668} \qquad \because z = \frac{52 - 40}{8} = 1.50$$

- Find the probability that an application takes between 36 and 48 minutes.

$$P(36 \leq X \leq 48) = P(-0.50 \leq Z \leq 1) = P(Z \leq 1) - P(Z \leq -0.50)$$
$$\simeq 0.8413 - 0.3085 = \boxed{0.5328}$$
$$\because z_1 = \frac{36 - 40}{8} = -0.50 \qquad z_2 = \frac{48 - 40}{8} = 1$$

- The company promises to finish applications within 48 minutes. What proportion of applications will meet this promise?

$$P(X \leq 48) = P(Z \leq 1) \simeq \boxed{0.8413} \qquad \because z = \frac{48 - 40}{8} = 1$$

  - Approximately 84.13% of applications will meet the promise.

- Another processing system has normally distributed processing times with the same mean but a standard deviation of 4 minutes. Under this system, would a greater or smaller proportion of applications meet the 48-minute promise? Explain without calculating.
  - A greater proportion would meet the promise. The smaller standard deviation concentrates processing times more closely around the mean of 40 minutes, reducing the proportion that exceeds 48 minutes.

Problem 3. Short Answers (continued)

3.F. An airline estimates that each passenger has an 80% probability of checking in online. Suppose 100 passengers independently decide whether to check in online. Let X be the number who check in online.

- Explain why a binomial distribution is appropriate. Identify n and p.
  - There is a fixed number of 100 independent trials. Each passenger either checks in online or does not, and every passenger has the same probability of checking in online.

$$\boxed{n = 100, \quad p = 0.80}$$

- Check whether the normal approximation is appropriate.

$$np = 100(0.80) = \boxed{80}$$
$$n(1-p) = 100(0.20) = \boxed{20}$$

  - Both values are at least 5, so the normal approximation is appropriate.

- Calculate the mean and standard deviation of X.

$$\mu = np = 100(0.80) = \boxed{80}$$
$$\sigma = \sqrt{np(1-p)} = \sqrt{100(0.80)(0.20)} = \sqrt{16} = \boxed{4}$$

- Using the normal approximation and the approach shown in class, estimate the probability that fewer than 72 passengers check in online.
  - Let N denote the approximating normal random variable with $\mu = 80$ and $\sigma = 4$.

$$P(X < 72) \simeq P(N < 72) = P(Z < -2) \simeq \boxed{0.0228} \qquad \because z = \frac{72 - 80}{4} = -2$$

- Estimate the probability that between 76 and 84 passengers, inclusive, check in online using the same normal approximation approach.

$$P(76 \le X \le 84) \simeq P(76 \le N \le 84) = P(Z \le 1) - P(Z \le -1)$$
$$\simeq 0.8413 - 0.1587 = \boxed{0.6826}$$
$$\because z_1 = \frac{76 - 80}{4} = -1 \qquad z_2 = \frac{84 - 80}{4} = 1$$