CONCORDIA COLLEGE
MOORHEAD, MINNESOTA

- Name: _____
- Date: _____

BUSN 301: Intermediate Microeconomic Theory

Quiz #3

Spring 2026: Suggested Solutions

INSTRUCTIONS:

- Each quiz is graded on a 100-point basis and contributes to your Quiz component of the course grade.
- You are expected to show all relevant steps and reasoning.
- Answers must be clearly written and well-organized.
- Graphs, when required, must be clearly labeled, with axes, curves, and key points identified.

Problem 1. Technology **(5 Points Each)**

Suppose that a firm's production function is given by:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

1.A. Compute the marginal product of each input.

$$MP_1 \equiv \frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}$$
$$MP_2 \equiv \frac{\partial f(x_1, x_2)}{\partial x_2} = \frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}$$

1.B. Find the marginal rate of technical substitution ($MRTS_{1,2}$).

$$MRTS_{1,2} = \frac{MP_1}{MP_2} = \frac{\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}}{\frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}} = \frac{x_2}{x_1}$$

1.C. Does this production function exhibit diminishing marginal product in each input? Briefly explain.

- Yes. We show the results for input 1, but the production function is symmetric.

$$\frac{\partial^2 f}{(\partial x_1)^2} = \frac{dMP_1}{dx_1} = -\frac{1}{4} x_1^{-\frac{3}{2}} x_2^{\frac{1}{2}} < 0$$

1.D. Does this production function exhibit increasing, constant, or decreasing returns to scale? Show your work.

- The production function exhibits constant returns to scale. For any $\lambda > 1$:

$$f(\lambda x_1, \lambda x_2) = (\lambda x_1)^{\frac{1}{2}} (\lambda x_2)^{\frac{1}{2}} = \lambda^{\frac{1}{2}} x_1^{\frac{1}{2}} \lambda^{\frac{1}{2}} x_2^{\frac{1}{2}} = \lambda x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = \lambda f(x_1, x_2)$$

1.E `(Extra Credit:2 Points)` If the firm's production function is $g(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{2}{3}}$, does g exhibit increasing, constant, or decreasing returns to scale?

- The production function exhibits increasing returns to scale. For any $\lambda > 1$:

$$g(\lambda x_1, \lambda x_2) = (\lambda x_1)^{\frac{2}{3}} (\lambda x_2)^{\frac{2}{3}} = \lambda^{\frac{2}{3}} x_1^{\frac{2}{3}} \lambda^{\frac{2}{3}} x_2^{\frac{2}{3}} = \lambda^{\frac{4}{3}} x_1^{\frac{2}{3}} x_2^{\frac{2}{3}} = \lambda^{\frac{4}{3}} g(x_1, x_2) > \lambda g(x_1, x_2)$$

Problem 2. Cost Minimization **(5 Points Each)**

Consider the same production function:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

Input prices are given by $w_1 = 4$ and $w_2 = 1$. The firm wishes to produce $\bar{y} = 4$ units of output.

2.A. Set up the firm's cost minimization problem.

$$\min_{x_1, x_2} 4x_1 + x_2 \quad s.t. \quad x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 4$$

2.B. Derive the condition that characterizes the optimal choice of inputs.

$$MRTS_{1,2} = \frac{\omega_1}{\omega_2} \quad \Rightarrow \quad \frac{\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}}{\frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}} = 4 \quad \Rightarrow \quad \frac{x_2}{x_1} = 4 \quad \Rightarrow \quad x_2 = 4x_1$$

2.C. Solve for the optimal inputs x_1^* and x_2^*.

$$4 = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = x_1^{\frac{1}{2}} (4x_1)^{\frac{1}{2}} = x_1^{\frac{1}{2}} \cdot 4^{\frac{1}{2}} \cdot x_1^{\frac{1}{2}} \quad \Rightarrow \quad 4 = \sqrt{4} x_1 \quad \Rightarrow \quad x_1^* = 2$$
$$\Rightarrow \quad x_2^* = 8$$

2.D. Compute the firm's minimum cost.

$$c(w_1, w_2, \bar{y}) = w_1 x_1^* + w_2 x_2^* \quad \Rightarrow \quad c(4, 1, 4) = 4 \cdot 2 + 1 \cdot 8 = 16$$

2.E. `(Extra Credit:2 Points)` Suppose that w_1 increases. How does this affect the ratio $\frac{x_1^*}{x_2^*}$? Briefly explain.

- If w_1 increases, then $\frac{x_1^*}{x_2^*}$ decreases, as cost minimization requires:

$$\frac{x_1^*}{x_2^*} = \frac{w_2}{w_1}$$

Problem 3. Cost Curves **(5 Points Each)**

Suppose that a firm's total cost function is given by:

$$c(y) = y^2 + 6y + 9$$

3.A. Identify the fixed cost and variable cost.

$$FC = 9, \qquad VC = y^2 + 6y$$

3.B. Derive the marginal cost (MC), average variable cost (AVC), and average total cost (ATC).

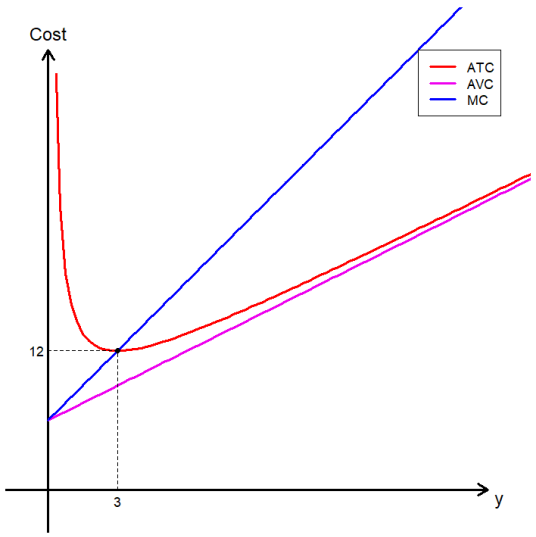
$$MC(y) \equiv \frac{dTC}{dy} = 2y + 6, \qquad AVC(y) \equiv \frac{VC}{y} = y + 6, \qquad ATC(y) \equiv \frac{TC}{y} = y + 6 + \frac{9}{y}$$

3.C. Find the output level that minimizes ATC. At what output level does MC intersect ATC?

$$\frac{dATC}{dy} = 0 \quad \Rightarrow \quad 1 - \frac{9}{y^2} = 0 \quad \Rightarrow \quad y^2 = 9 \quad \Rightarrow \quad y = 3$$

3.D. Graph the MC, AVC, and ATC curves. Clearly label:

- The minimum of ATC
- The intersection of MC and ATC
- The relative positions of AVC and ATC



Problem 4. Firm Supply **(5 Points Each)**

Suppose that a competitive firm's total cost function is:

$$c(y) = y^2 + 6y + 9$$

4.A. Find the first-order condition for the firm's profit maximization.

$$\frac{d\pi}{dy} = 0 \quad \Rightarrow \quad \frac{d}{dy}\left(p \cdot y - C(y)\right) = 0 \quad \Rightarrow \quad p - MC(y) = 0 \quad \Rightarrow \quad p - 2y - 6 = 0$$
$$\Rightarrow \quad y^* = \frac{p-6}{2}$$

4.B. Find the firm's short-run supply function.

- The supply curve is the MC that is increasing in y, which is true for all values of y.
- The supply curve is the portion of the MC curve that lies above the AVC curve.

$$MC \geq AVC \quad \Rightarrow \quad 2y + 6 \geq y + 6 \quad \Rightarrow \quad y \geq 0$$

- So, firm i's supply function is:

$$S_i(p) = \begin{cases} \frac{p-6}{2}, & p \geq 6 \\ 0, & \text{otherwise} \end{cases}$$

4.C. What is the shutdown price?

- The firm shuts down in the short run when $p < \min AVC = 6$.

4.D. Suppose that the market price is $p = 10$. Find the firm's optimal output and profit.

- According to the supply function, the firm's optimal output is 2 units.
- The firm's profit is then:

$$\pi = 10 \cdot 2 - \left(2^2 + 6 \cdot 2 + 9\right) = -5$$

Problem 5. Industry Supply **(5 Points Each)**

Suppose there are n identical firms in the market, each with the cost function from Problem 4. Market demand is given by:

$$D(p) = 62 - p$$

5.A. Derive the industry supply function.

- Since all firms are identical:

$$S(p) = \sum_{i=1}^{n} S_i(p) \quad \Rightarrow \quad S(p) = \begin{cases} \frac{n(p-6)}{2}, & p \geq 6 \\ 0, & \text{otherwise} \end{cases}$$

5.B. Suppose that $n = 5$. Find the market equilibrium price and quantity.

$$S(p^*) = D(p^*) \quad \Rightarrow \quad \frac{5(p^* - 6)}{2} = 62 - p^* \quad \Rightarrow \quad 7p^* = 154 \quad \Rightarrow \quad p^* = 22$$
$$\Rightarrow \quad q^* = 40$$

5.C. At this equilibrium, do firms earn positive, zero, or negative profit? Briefly explain.

- Since there are 5 firms in the market, each firm's output is 8 units.
- The firm's profit is then:

$$\pi = 22 \cdot 8 - \left(8^2 + 6 \cdot 8 + 9\right) = 55$$

5.D. Describe what happens in the long run if firms are earning positive profit.

- Since firms earn positive profit, new firms will enter the market.
- Entry increases industry supply, shifting the supply curve outward.
- As supply increases, the market price falls.
- This process continues until firms earn zero profit in the long run.

5.E. `(Extra Credit:2 Points)` Find the long-run equilibrium price.

- In the long run with free entry and exit, identical technologies, and no market power, the market price of the output converges to min $ATC = 12$.

Problem 6. Feedback `(Extra Credit:1 Point Each)`

6.A. Which chapter(s) that were covered in this quiz did you find challenging? (Select all that apply.)

- Chapter 19: Technology (e.g., marginal product, MRTS)
- Chapter 20: Profit Maximization (e.g., profit function, first-order condition)
- Chapter 21: Cost Minimization (e.g., short run vs. long run, input choice)
- Chapter 22: Cost Curves (e.g., MC, AVC, ATC)
- Chapter 23: Firm Supply (e.g., shutdown condition, profit)
- Chapter 24: Industry Supply (e.g., entry, long-run equilibrium)

Briefly explain why you found this part challenging (1–2 sentences).

6.B. At this point in the course, which statement best describes you?

- I understand the ideas conceptually but struggle with the math
- I can do the math but do not always understand what it means
- I am comfortable with both the math and the intuition
- I feel lost and am not sure where my confusion starts

Briefly say what would help most in class.

- Original Score: _____
- Original Date: _____
- Recovered Score: _____
- Recovered Date: _____