CONCORDIA COLLEGE
MOORHEAD, MINNESOTA

- Name: _____
- Date: _____

BUSN 301: Intermediate Microeconomic Theory

Problem Set #4

Spring 2026

INSTRUCTIONS:

- Each problem set is graded on a 100-point basis and contributes to your Problem Set component of the course grade.
- You are expected to show all relevant steps and reasoning.
- Answers must be clearly written and well-organized.
- Graphs, when required, must be clearly labeled, with axes, curves, and key points identified.
- Problem sets must be submitted by the posted deadline.

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Problem 1. Exchange

Consider a pure exchange economy with two agents, A and B, and two goods, 1 and 2. Each agents' utility function is given by:

$$u_A(x_A^1, x_A^2) = (x_A^1)^{\frac{1}{2}}(x_A^2)^{\frac{1}{2}}, \qquad u_B(x_B^1, x_B^2) = (x_B^1)^{\frac{1}{4}}(x_B^2)^{\frac{3}{4}}$$

The initial endowment of the economy is given by $(\omega_A^1, \omega_A^2, \omega_B^1, \omega_B^2) = (4, 2, 2, 6)$.

1.A. State the utility maximization problem for agent A. Clearly define the objective function and their budget constraint.

1.B. Solve agent A's utility maximization problem to obtain their demand for goods 1 and 2 as functions of prices (p_1, p_2) and income.

1.C. Repeat parts `1.A` and `1.B` for agent B.

Problem 1. Exchange (continued)

1.D. Let good 2 be the numeraire good, so that $p_2 = 1$, and let $p_1 = p$. Compute each agent's income as a function of p.

1.E. Substitute income into each agent's demand for good 1. Express demand as a function of p.

1.F. Find the competitive equilibrium price p^*, and the equilibrium allocation.

1.G. Verify that the equilibrium is Pareto optimal (brief explanation).

Problem 2. Production

Consider an economy with one consumer, one firm, and two goods, 1 and 2. The consumer's initial endowment is given by $(\omega^1, \omega^2) = (0, 12)$. The consumer's utility function is given by

$$u(x^1, x^2) = (x^1)^{\frac{1}{2}}(x^2)^{\frac{1}{2}}.$$

The firm uses good 2 as an input to produce good 1 according to the production function

$$y = f(z) = z^{\frac{1}{2}},$$

where z denotes the amount of good 2 used as input, and y denotes the amount of good 1 produced. Assume that the consumer owns the firm.

2.A. State the firm's profit maximization problem. Clearly define the firm's profit function.

2.B. Solve the firm's profit maximization problem to find its demand for input z, output supply y, and profit as functions of prices (p_1, p_2).

2.C. State the consumer's utility maximization problem. Clearly define the consumer's budget constraint.

Problem 2. Production (continued)

2.D. Let good 2 be the numeraire good, so that $p_2 = 1$, and let $p_1 = p$. Express the consumer's income as a function of p, taking into account both the value of the initial endowment and firm profit.

2.E. Solve the consumer's utility maximization problem to find the consumer's demand for good 1 as a function of p.

2.F. Find the competitive equilibrium price p^*.

Problem 2. Production (continued)

2.G. Compute the competitive equilibrium allocation and the firm's equilibrium production plan.

2.H. Verify that, at equilibrium, the consumer's marginal rate of substitution equals the marginal rate of transformation.

Problem 3. Welfare

Consider the pure exchange economy described in Problem 1.

3.A. Define what it means for an allocation to be Pareto optimal.

3.B. Define what it means for an allocation to be envy-free.

3.C. Define what it means for an allocation to be fair.

3.D. Is the competitive equilibrium allocation from Problem 1 guaranteed to be Pareto optimal? Briefly explain.

Problem 3. Welfare (continued)

3.E. Suppose instead that the initial endowment is equally split: $(\omega_A^1, \omega_A^2, \omega_B^1, \omega_B^2) = (3, 4, 3, 4)$. Would the competitive equilibrium allocation be considered fair? Briefly explain.

3.F. Consider the social welfare function $W(u_A, u_B) = u_A + u_B$. Describe the problem faced by a social planner. What condition must hold at the optimal allocation?

3.G. Briefly explain the Second Welfare Theorem and its economic intuition.

- Score: _____
- Date: _____