



• Name: _____

• Date: _____

BUSN 301: Intermediate Microeconomic Theory

Problem Set #3

Spring 2026

INSTRUCTIONS:

- Each problem set is graded on a 100-point basis and contributes to your Problem Set component of the course grade.
- You are expected to show all relevant steps and reasoning.
- Answers must be clearly written and well-organized.
- Graphs, when required, must be clearly labeled, with axes, curves, and key points identified.
- Problem sets must be submitted by the posted deadline.

Problem 1. The Production Function

Suppose that a producer's production technology is represented by the following production function:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

1.A. Plot and compare the isoquants corresponding to $\bar{y} = 8$ and $\bar{y} = 16$.

1.B. Find the expression for the marginal product of each factor of production.

1.C. In your own words, explain what the marginal product of each factor measures.

1.D. Show that the marginal product of each input is decreasing in its own input (holding the other input fixed). Does this confirm the law of diminishing marginal product?

Problem 1. The Production Function (continued)

1.E. Find the expression for the marginal rate of technical substitution between inputs 1 and 2.

1.F. Based on your graph in 1.A, how can the marginal rate of technical substitution be interpreted visually? Explain your answer.

1.G. Show that the marginal rate of technical substitution is diminishing as input 1 increases (holding output constant).

1.H. Determine whether the production function exhibits increasing, constant, or decreasing returns to scale. Show your work by evaluating $f(\lambda x_1, \lambda x_2)$ for $\lambda > 0$, and comparing it to $\lambda f(x_1, x_2)$.

Problem 2. Cost Minimization in the Long Run

Consider a firm with the following production function:

$$f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$$

The prices of inputs are given by w_1 and w_2 , and the firm wishes to produce $\bar{y} > 0$ units of output.

2.A. Set up the firm's cost minimization problem.

2.B. Derive the condition that characterizes the firm's optimal choice of inputs. Interpret this condition in economic terms.

2.C. Illustrate the firm's cost minimization problem graphically. In your diagram, include:

- An isoquant corresponding to \bar{y}
- An isocost line
- The optimal bundle of inputs

Explain how the optimal choice is determined in your graph.

Problem 2. Cost Minimization in the Long Run (continued)

2.D. Solve for the firm's conditional factor demands $x_1^*(w_1, w_2, \bar{y})$ and $x_2^*(w_1, w_2, \bar{y})$.

2.E. Derive the firm's cost function $C(w_1, w_2, \bar{y})$.

2.F. In your own words, explain how changes in input prices affect the firm's optimal choice of inputs.

2.G. Suppose that w_1 increases while w_2 remains constant. How does this affect the ratio $\frac{x_1^*}{x_2^*}$? Provide intuition for your answer.

Problem 3. Cost Minimization in the Short Run

Consider the same production function:

$$f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$$

Suppose that input 2 is fixed in the short run at $\bar{x}_2 > 0$, while input 1 remains variable.

3.A. Write down the firm's short-run cost minimization problem.

3.B. Solve for the firm's optimal choice of x_1 as a function of \bar{y} and \bar{x}_2 .

3.C. Derive the short-run cost function $C^{SR}(w_1, \bar{x}_2, \bar{y})$.

3.D. Explain why the condition $MRTS_{12} = \frac{w_1}{w_2}$ generally does not hold in the short run.

Problem 3. Cost Minimization in the Short Run (continued)

3.E. Illustrate the firm's short-run cost minimization problem graphically. In your diagram, include:

- An isoquant corresponding to \bar{y}
- An isocost line
- The fixed level of input 2
- The firm's optimal choice of inputs

Explain how this differs from the long-run case.

3.F. Compare the short-run cost function to the long-run cost function derived in Problem 2. Which one is higher for a given level of output? Explain why.

3.G. Suppose the fixed input level \bar{x}_2 is very small. How does this affect the firm's short-run cost? Provide intuition.

Problem 4. Cost Curves

Suppose that a firm's total cost function is given by:

$$C(y) = 5y^2 + 3y + 45$$

- 4.A. Identify the firm's fixed cost, variable cost, and total cost.
- 4.B. Derive the firm's marginal cost (MC), average variable cost (AVC), and average total cost (ATC).
- 4.C. At what level of output is average total cost minimized? Show your work.
- 4.D. Verify that marginal cost equals average total cost at the minimum of the ATC curve.

Problem 4. Cost Curves (continued)

4.E. In your own words, explain why the marginal cost curve intersects the ATC curve at its minimum.

4.F. Explain why the ATC curve always lies above the AVC curve.

4.G. Graph the MC, AVC, and ATC curves. Clearly label:

- The minimum of the ATC curve
- The point where MC intersects ATC
- The relative positions of AVC and ATC

Problem 5. Firm Supply

Suppose that a competitive firm's total cost function is given by:

$$C(y) = y^2 + 4y + 9$$

- 5.A. Find the firm's marginal cost, average variable cost, and average total cost.
- 5.B. Find the output level that satisfies the firm's first-order condition for profit maximization at price p .
- 5.C. Determine the firm's short-run supply function.
- 5.D. At what price will the firm shut down in the short run? Explain why the shutdown condition depends on average variable cost rather than average total cost.

Problem 5. Firm Supply (continued)

5.E. Suppose that the market price is $p = 10$. Find the firm's optimal output and profit.

5.F. Suppose that the market price is $p = 10$. Find the firm's producer surplus.

5.G. Illustrate the firm's producer surplus graphically. In your diagram, include:

- The firm's marginal cost curve
- The market price
- The firm's chosen output
- The region corresponding to producer surplus

Problem 6. Industry Supply

Suppose that there are n identical competitive firms in a market. Each firm has the total cost function:

$$C(y) = y^2 + 4y + 9$$

6.A. Using your results from Problem 5, write down the firm's short-run supply function.

6.B. Derive the short-run industry supply function as a function of n and p .

6.C. Suppose that market demand is given by:

$$D(p) = 100 - p$$

Find the short-run equilibrium price as a function of n .

Problem 6. Industry Supply (continued)

6.D. Suppose that $n = 10$. Find the short-run equilibrium price and total quantity in the market.

6.E. At the equilibrium price found in part 6 . D, determine whether firms earn positive, zero, or negative profit. Show your calculations.

6.F. Based on your answer in part 6 . E, describe what will happen in the long run. Explain how entry or exit affects industry supply and market price.

6.G. In the long run, what condition must hold for firms in equilibrium? Using this condition, solve for the long-run equilibrium price.

• Score: _____

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