CONCORDIA COLLEGE
MOORHEAD, MINNESOTA

- Name: ______________________________
- Date: ______________________________

BUSN 301: Intermediate Microeconomic Theory

Problem Set #3: Suggested Solutions

Spring 2026

INSTRUCTIONS:

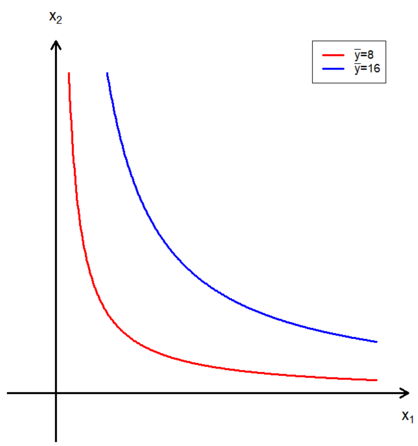
- Each problem set is graded on a 100-point basis and contributes to your Problem Set component of the course grade.
- You are expected to show all relevant steps and reasoning.
- Answers must be clearly written and well-organized.
- Graphs, when required, must be clearly labeled, with axes, curves, and key points identified.
- Problem sets must be submitted by the posted deadline.

Problem 1. The Production Function

Suppose that a producer's production technology is represented by the following production function:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

1.A. Plot and compare the isoquants corresponding to $\bar{y} = 8$ and $\bar{y} = 16$.



- To find the equation of the isoquant for some level of output $\bar{y}$:

$$\bar{y} = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} \quad \Rightarrow \quad x_2^{\frac{1}{2}} = \frac{\bar{y}}{x_1^{\frac{1}{2}}}$$

$$\Rightarrow \quad x_2 = \left(\frac{\bar{y}}{x_1^{\frac{1}{2}}} \right)^2$$

1.B. Find the expression for the marginal product of each factor of production.

$$MP_1 \equiv \frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$MP_2 \equiv \frac{\partial f(x_1, x_2)}{\partial x_2} = \frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}$$

1.C. In your own words, explain what the marginal product of each factor measures.

- Roughly, the marginal product is a measure of how much additional output is produced when the firm uses a little more of a given input.
- More precisely, the marginal product of an input is the increase in output generated by a one-unit increase in that input, holding the other input fixed.

1.D. Show that the marginal product of each input is decreasing in its own input (holding the other input fixed). Does this confirm the law of diminishing marginal product?

- Yes. We can check this by verifying that the second derivative of the production function with respect to each input is negative.

$$\frac{\partial^2 f}{(\partial x_1)^2} = \frac{dMP_1}{dx_1} = -\frac{1}{4} x_1^{-\frac{3}{2}} x_2^{\frac{1}{2}} < 0$$

Problem 1. The Production Function (continued)

1.E. Find the expression for the marginal rate of technical substitution between inputs 1 and 2.

$$MRTS_{1,2} = \frac{MP_1}{MP_2} = \frac{\frac{1}{2}x_1^{-\frac{1}{2}}x_2^{\frac{1}{2}}}{\frac{1}{2}x_1^{\frac{1}{2}}x_2^{-\frac{1}{2}}} = \frac{x_2}{x_1}$$

1.F. Based on your graph in `1.A`, how can the marginal rate of technical substitution be interpreted visually? Explain your answer.

- Graphically, the marginal rate of technical substitution is represented by the slope of the isoquant at a given point.
- Economically, it measures how much of input 2 the firm can give up when it uses one more unit of input 1, while holding output constant.

1.G. Show that the marginal rate of technical substitution is diminishing as input 1 increases.

$$\frac{\partial MRTS_{1,2}}{\partial x_1} = \frac{\partial}{\partial x_1}\left(\frac{x_2}{x_1}\right) = -\frac{x_2}{x_1^2} < 0$$

1.H. Determine whether the production function exhibits increasing, constant, or decreasing returns to scale. Show your work by evaluating $f(\lambda x_1, \lambda x_2)$ for $\lambda > 0$, and comparing it to $\lambda f(x_1, x_2)$.

- The production function exhibits constant returns to scale:

$$f(\lambda x_1, \lambda x_2) = (\lambda x_1)^{\frac{1}{2}}(\lambda x_2)^{\frac{1}{2}} = \lambda^{\frac{1}{2}}x_1^{\frac{1}{2}}\lambda^{\frac{1}{2}}x_2^{\frac{1}{2}} = \lambda x_1^{\frac{1}{2}}x_2^{\frac{1}{2}} = \lambda f(x_1, x_2)$$

Problem 2. Cost Minimization in the Long Run

Consider a firm with the following production function:

$$f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$$

The prices of inputs are given by w_1 and w_2, and the firm wishes to produce $\bar{y} > 0$ units of output.

2.A. Set up the firm's cost minimization problem.

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2 \quad s.t. \quad x_1^{\frac{1}{4}} x_2^{\frac{3}{4}} = \bar{y}$$

2.B. Derive the condition that characterizes the firm's optimal choice of inputs. Interpret this condition in economic terms.

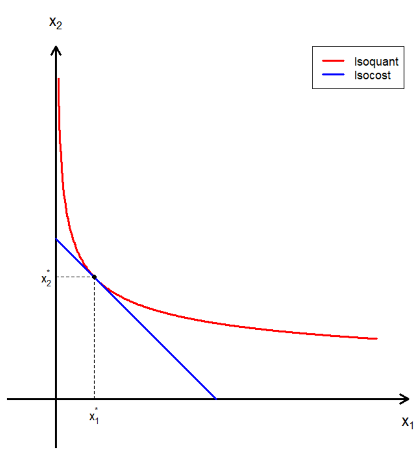
$$MRTS_{1,2} = \frac{\omega_1}{\omega_2} \quad \Rightarrow \quad \frac{\frac{1}{4} x_1^{-\frac{3}{4}} x_2^{\frac{3}{4}}}{\frac{3}{4} x_1^{\frac{1}{4}} x_2^{-\frac{1}{4}}} = \frac{w_1}{w_2} \quad \Rightarrow \quad \frac{x_2}{3x_1} = \frac{w_1}{w_2} \quad \Rightarrow \quad x_2 = \frac{3w_1}{w_2} x_1$$

- At the cost-minimizing bundle, the slope of the isoquant equals the slope of the isocost line. The firm chooses inputs so that the rate at which technology allows substitution between inputs equals the rate at which the market allows substitution through input prices.

2.C. Illustrate the firm's cost minimization problem graphically. In your diagram, include:

- An isoquant corresponding to $\bar{y}$
- An isocost line
- The optimal bundle of inputs

Explain how the optimal choice is determined in your graph.



- The optimal bundle occurs at the tangency point between the isoquant and the isocost line.
- At this point, the slope of the isoquant equals the slope of the isocost line. Therefore:

$$MRTS_{1,2} = \frac{w_1}{w_2}$$

Problem 2. Cost Minimization in the Long Run (continued)

2.D. Solve for the firm's conditional factor demands $x_1^*(w_1, w_2, \bar{y})$ and $x_2^*(w_1, w_2, \bar{y})$.

$$\bar{y} = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}} = x_1^{\frac{1}{4}} \left(\frac{3w_1}{w_2} x_1\right)^{\frac{3}{4}} = x_1^{\frac{1}{4}} \left(\frac{3w_1}{w_2}\right)^{\frac{3}{4}} x_1^{\frac{3}{4}} \quad \Rightarrow \quad \bar{y} = \left(\frac{3w_1}{w_2}\right)^{\frac{3}{4}} x_1$$

$$\Rightarrow \quad x_1^*(w_1, w_2, \bar{y}) = \left(\frac{w_2}{3w_1}\right)^{\frac{3}{4}} \bar{y}$$

$$\Rightarrow \quad x_2^*(w_1, w_2, \bar{y}) = \left(\frac{3w_1}{w_2}\right)^{\frac{1}{4}} \bar{y}$$

2.E. Derive the firm's cost function $C(w_1, w_2, \bar{y})$.

$$C(w_1, w_2, \bar{y}) = w_1 x_1^* + w_2 x_2^* = w_1 \left(\frac{w_2}{3w_1}\right)^{\frac{3}{4}} \bar{y} + w_2 \left(\frac{3w_1}{w_2}\right)^{\frac{1}{4}} \bar{y}$$

$$\Rightarrow \quad C(w_1, w_2, \bar{y}) = \frac{4}{3^{3/4}} \, w_1^{1/4} w_2^{3/4} \bar{y}$$

2.F. In your own words, explain how changes in input prices affect the firm's optimal choice of inputs.

- When the price of an input rises relative to the other input, the firm substitutes away from the relatively more expensive input and toward the relatively cheaper input.
- In this problem, the cost-minimizing bundle satisfies $x_2 = \frac{3w_1}{w_2} x_1$, so changes in input prices affect the optimal ratio of inputs.
- Because the exponent on input 2 is larger, output is more sensitive to input 2 in this production function.

2.G. Suppose that w_1 increases while w_2 remains constant. How does this affect the ratio $\frac{x_1^*}{x_2^*}$? Provide intuition for your answer.

- If w_1 increases while w_2 remains constant, then $\frac{x_1^*}{x_2^*}$ decreases.
- Intuitively, as input 1 becomes more expensive, the firm substitutes away from input 1 and uses relatively more input 2.

Problem 3. Cost Minimization in the Short Run

Consider the same production function:

$$f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$$

Suppose that input 2 is fixed in the short run at $\bar{x}_2 > 0$, while input 1 remains variable.

3.A. Write down the firm's short-run cost minimization problem.

$$\min_{x_1} w_1 x_1 + w_2 \bar{x}_2 \quad s.t. \quad x_1^{\frac{1}{4}} \bar{x}_2^{\frac{3}{4}} = \bar{y}$$

3.B. Solve for the firm's optimal choice of x_1 as a function of $\bar{y}$ and $\bar{x}_2$.

$$x_1^{\frac{1}{4}} \bar{x}_2^{\frac{3}{4}} = \bar{y} \quad \Rightarrow \quad x_1^{\frac{1}{4}} = \bar{y} \bar{x}_2^{-\frac{3}{4}} \quad \Rightarrow \quad x_1^*(\bar{y}, \bar{x}_2) = \frac{\bar{y}^4}{\bar{x}_2^3}$$

3.C. Derive the short-run cost function $C^{SR}(w_1, \bar{x}_2, \bar{y})$.

$$C^{SR}(w_1, \bar{x}_2, \bar{y}) = w_1 x_1^* + w_2 \bar{x}_2 = w_1 \frac{\bar{y}^4}{\bar{x}_2^3} + w_2 \bar{x}_2$$

3.D. Explain why the condition $MRTS_{12} = \frac{w_1}{w_2}$ generally does not hold in the short run.

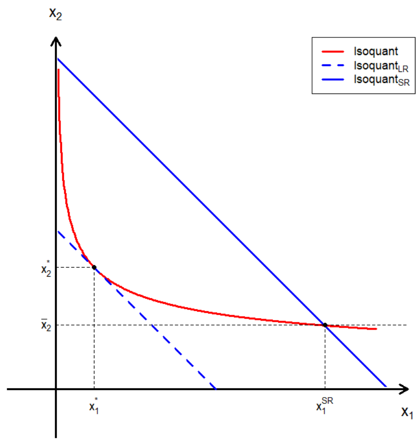
- In the long run, the firm can freely adjust both inputs, so the cost-minimizing bundle occurs at a tangency between the isoquant and the isocost line.
- In the short run, input 2 is fixed at $\bar{x}_2$, so the firm cannot freely choose both inputs.
- Therefore, the firm is generally not at a tangency point, and the condition $MRTS_{1,2} = \frac{w_1}{w_2}$ does not generally hold.
- Instead, the firm chooses the lowest-cost feasible point along the vertical line $x_2 = \bar{x}_2$.

Problem 3. Cost Minimization in the Short Run (continued)

3.E. Illustrate the firm's short-run cost minimization problem graphically. In your diagram, include:

- An isoquant corresponding to $\bar{y}$
- An isocost line
- The fixed level of input 2
- The firm's optimal choice of inputs

Explain how this differs from the long-run case.



- In the short run, input 2 is fixed at $x_2 = \bar{x}_2$.
- The firm chooses the lowest-cost point along the constraint $x_2 = \bar{x}_2$.
- The optimal bundle occurs at the intersection of the isoquant and the line $x_2 = \bar{x}_2$.
- This differs from the long-run case, where the optimal bundle occurs at a tangency between the isoquant and the isocost line.

3.F. Compare the short-run cost function to the long-run cost function derived in Problem 2. Which one is higher for a given level of output? Explain why.

$$C^{SR}(w_1, \bar{x}_2, \bar{y}) \geq C(w_1, w_2, \bar{y})$$

- This is because in the short run the firm faces an additional constraint: input 2 is fixed and cannot be adjusted freely.
- In the long run, the firm can choose both inputs optimally, so it can always achieve the same output at lower cost.

3.G. Suppose the fixed input level $\bar{x}_2$ is very small. How does this affect the firm's short-run cost? Provide intuition.

- Input 2 is especially important because it has the larger exponent: $f(x_1, x_2) = x_1^{1/4} x_2^{3/4}$.
- To produce a given level of output $\bar{y}$, the firm must use a large amount of $x_1^* = \frac{\bar{y}^4}{\bar{x}_2^3}$
- Therefore, short-run cost becomes very high when $\bar{x}_2$ is very small.
- Intuitively, when the fixed input is scarce, the firm must compensate by using much more of the variable input.

Problem 4. Cost Curves

Suppose that a firm's total cost function is given by:

$$C(y) = 5y^2 + 3y + 45$$

4.A. Identify the firm's fixed cost, variable cost, and total cost.

$$FC = 45$$
$$VC = 5y^2 + 3y$$
$$TC = 5y^2 + 3y + 45$$

4.B. Derive the firm's marginal cost (MC), average variable cost (AVC), and average total cost (ATC).

$$AVC(y) \equiv \frac{VC}{y} = 5y + 3$$
$$ATC(y) \equiv \frac{TC}{y} = 5y + 3 + \frac{45}{y}$$
$$MC(y) \equiv \frac{dTC}{dy} = 10y + 3$$

4.C. At what level of output is average total cost minimized? Show your work.

$$\frac{dATC}{dy} = 0 \quad \Rightarrow \quad 5 - \frac{45}{y^2} = 0 \quad \Rightarrow \quad y^2 = 9 \quad \Rightarrow \quad y = 3$$

4.D. Verify that marginal cost equals average total cost at the minimum of the ATC curve.

- At the ATC minimizing output $y = 3$, the average total cost is $ATC = 5 \cdot 3 + 3 + \frac{45}{3} = 33$.
- When $y = 3$, the marginal cost is $MC = 10 \cdot 3 + 3 = 33$.

Problem 4. Cost Curves (continued)

4.E. In your own words, explain why the marginal cost curve intersects the ATC curve at its minimum.

- When MC ¡ ATC, producing an additional unit lowers the average total cost, so ATC is decreasing.
- When MC ¿ ATC, producing an additional unit raises the average total cost, so ATC is increasing.
- Therefore, the point where MC = ATC corresponds to the minimum of the ATC curve.

4.F. Explain why the ATC curve always lies above the AVC curve.

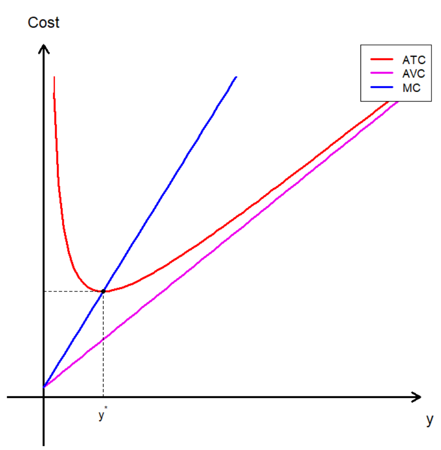
- Total cost is the sum of fixed and variable costs, which are both non-negative.

$$TC = FC + VC \quad \Rightarrow \quad \frac{TC}{y} = \frac{FC}{y} + \frac{VC}{y} \quad \Rightarrow \quad ATC = AFC + AVC$$

- Since average fixed cost (AFC) is always positive, ATC must always lie above AVC.

4.G. Graph the MC, AVC, and ATC curves. Clearly label:

- The minimum of the ATC curve
- The point where MC intersects ATC
- The relative positions of AVC and ATC



- The MC, AVC, and ATC curves are all upward sloping.
- The AVC curve lies below the ATC curve for all output levels.
- The ATC curve reaches its minimum at $y = 3$.
- The MC curve intersects the ATC curve at its minimum point.
- The MC curve also intersects the AVC curve at its minimum.

Problem 5. Firm Supply

Suppose that a competitive firm's total cost function is given by:

$$C(y) = y^2 + 4y + 9$$

5.A. Find the firm's marginal cost, average variable cost, and average total cost.

$$AVC(y) \equiv \frac{VC}{y} = \frac{y^2 + 4y}{y} = y + 4$$
$$ATC(y) \equiv \frac{TC}{y} = \frac{y^2 + 4y + 9}{y} = y + 4 + \frac{9}{y}$$
$$MC(y) \equiv \frac{dTC}{dy} = 2y + 4$$

5.B. Find the output level that satisfies the firm's first-order condition for profit maximization at price p.

$$\frac{d\pi}{dy} = 0 \quad \Rightarrow \quad \frac{d}{dy} p \cdot y - C(y) = 0 \quad \Rightarrow \quad p - MC(y) = 0 \quad \Rightarrow \quad p - 2y - 4 = 0$$
$$\Rightarrow \quad y^* = \frac{p-4}{2}$$

5.C. Determine the firm's short-run supply function.

- The supply curve is the MC that is increasing in y, which is true for all values of y.
- The supply curve is the portion of the MC curve that lies above the AVC curve.

$$MC \geq AVC \quad \Rightarrow \quad 2y + 4 \geq y + 4 \quad \Rightarrow \quad y \geq 0$$

- So, the firm's supply function is:

$$S(p) = \begin{cases} \frac{p-4}{2}, & p \geq 4 \\ 0, & \text{otherwise} \end{cases}$$

5.D. At what price will the firm shut down in the short run? Explain why the shutdown condition depends on average variable cost rather than average total cost.

- The firm shuts down if $p < \min AVC = 4$.
- In the short run, fixed costs are sunk and must be paid regardless of output. Therefore, the firm compares revenue only to variable cost.

Problem 5. Firm Supply (continued)

5.E. Suppose that the market price is $p = 10$. Find the firm's optimal output and profit.

- According to the supply function, the firm's optimal output is 3 units.
- The firm's profit is then:

$$\pi = 10 \cdot 3 - \left(3^2 + 4 \cdot 3 + 9\right) = 0$$

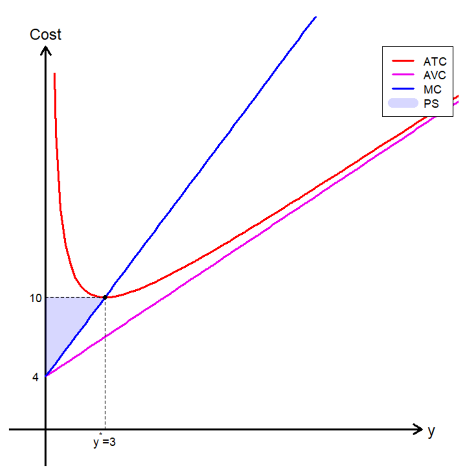
5.F. Suppose that the market price is $p = 10$. Find the firm's producer surplus.

- At $p = 10$, the firm chooses $y^* = 3$.
- Producer surplus is the area above (or, to the left of) the marginal cost curve, and below the market price, from $y = 0$ to $y^* = 3$.

$$PS = \frac{1}{2} \cdot (10 - 4) \cdot 3 = 9$$

5.G. Illustrate the firm's producer surplus graphically. In your diagram, include:

- The firm's marginal cost curve
- The market price
- The firm's chosen output
- The region corresponding to producer surplus



Problem 6. Industry Supply

Suppose that there are n identical competitive firms in a market. Each firm has the total cost function:

$$C(y) = y^2 + 4y + 9$$

6.A. Using your results from Problem 5, write down the firm's short-run supply function.

$$S_i(p) = \begin{cases} \frac{p-4}{2}, & p \geq 4 \\ 0, & \text{otherwise} \end{cases}$$

6.B. Derive the short-run industry supply function as a function of n and p.

- The industry supply curve is the horizontal sum of individual firm supply curves.
- Since all firms are identical:

$$S(p) = \sum_{i=1}^{n} S_i(p) \quad \Rightarrow \quad S(p) = \begin{cases} \frac{n(p-4)}{2}, & p \geq 4 \\ 0, & \text{otherwise} \end{cases}$$

6.C. Suppose that market demand is given by:

$$D(p) = 100 - p$$

Find the short-run equilibrium price as a function of n.

$$\begin{aligned} S(p) = D(p) \quad \Rightarrow \quad \frac{np - 4n}{2} = 100 - p \quad &\Rightarrow \quad \frac{np}{2} + p = \frac{4n}{2} + 100 \\ &\Rightarrow \quad p\left(\frac{n+2}{2}\right) = 2n + 100 \\ &\Rightarrow \quad p = \frac{2n+100}{\frac{n+2}{2}} \\ &\Rightarrow \quad p = \frac{4n+200}{n+2} \end{aligned}$$

- We can also verify that $\frac{4n+200}{n+2}$ is greater than or equal to 4 for all values of n, which is consistent with the industry supply function.

Problem 6. Industry Supply (continued)

6.D. Suppose that $n = 10$. Find the short-run equilibrium price and total quantity in the market.

- From `6.C`, we find that $p^* = \frac{240}{12} = 20$.
- Plugging in p^* to either $S(p)$ or $D(p)$, we find that $q^* = 80$.

6.E. At the equilibrium price found in part `6.D`, determine whether firms earn positive, zero, or negative profit. Show your calculations.

- Assuming that each of the 10 firms produce 8 units, we find that each firm's profit is:

$$\pi_i = 20 \cdot 8 - (8^2 + 4 \cdot 8 + 9) = 55$$

6.F. Based on your answer in part `6.E`, describe what will happen in the long run. Explain how entry or exit affects industry supply and market price.

- Since firms earn positive profit, new firms will enter the market.
- Entry increases industry supply, shifting the supply curve outward.
- As supply increases, the market price falls.
- This process continues until firms earn zero profit in the long run.

6.G. In the long run, what condition must hold for firms in equilibrium? Using this condition, solve for the long-run equilibrium price.

- Each firm earns $\pi_i = 0$ in the long run equilibrium, and this occurs when $p^* = \min ATC$:

$$\frac{dATC}{dy} = 1 - \frac{9}{y^2} = 0 \quad \Rightarrow \quad y = 3 \quad \Rightarrow \quad \min ATC = 10$$

- The long run equilibrium price is $p^* = 10$.

- Score: ______________________
- Date: ______________________