CONCORDIA COLLEGE
MOORHEAD, MINNESOTA

- Name: _____
- Date: _____

BUSN 301: Intermediate Microeconomic Theory

Problem Set #3: Optional

Spring 2026

INSTRUCTIONS:

- This problem set is ungraded and is provided as a supplemental resource to the course.

Problem 1. The Production Function

Suppose that a producer's production technology is represented by the following production function:

$$f(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$$

1.A. Plot and compare the isoquants corresponding to $\bar{y} = 4$ and $\bar{y} = 8$.

1.B. Find the expression for the marginal product of each factor of production.

1.C. Which input exhibits stronger diminishing marginal product? Explain using your expressions.

1.D. Find the expression for the marginal rate of technical substitution between inputs 1 and 2.

Problem 1. The Production Function (continued)

1.E. Determine whether the production function exhibits increasing, constant, or decreasing returns to scale. Show your work by evaluating $f(\lambda x_1, \lambda x_2)$ for $\lambda > 0$, and comparing it to $\lambda f(x_1, x_2)$.

1.F. Consider another production technology $g(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$. Plot the isoquants of f and g, each corresponding to $\bar{y} = 1$. (You may find it helpful to rewrite each isoquant as x_2 as a function of x_1.)

1.G. Compare the two isoquants. Which input is used more intensively in each production function? Explain your answer.

Problem 2. Cost Minimization in the Long Run

Consider a firm with the following production function:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

The prices of inputs are given by w_1 and w_2, and the firm wishes to produce $\bar{y} > 0$ units of output.

2.A. Set up the firm's cost minimization problem.

2.B. Derive the condition that characterizes the firm's optimal choice of inputs. Interpret this condition in economic terms.

2.C. Solve for the firm's conditional factor demands $x_1^*(w_1, w_2, \bar{y})$ and $x_2^*(w_1, w_2, \bar{y})$.

2.D. Derive the firm's cost function $C(w_1, w_2, \bar{y})$.

Problem 2. Cost Minimization in the Long Run (continued)

2.E. Suppose that $w_1 = w_2$. What does this imply about the firm's optimal choice of inputs? Explain your answer.

2.F. Illustrate the firm's cost minimization problem graphically. In your diagram, include:

- An isoquant corresponding to $\bar{y}$
- An isocost line
- The optimal bundle of inputs

Explain how the condition from part `2.B` is reflected in your graph.

Problem 3. Cost Minimization in the Short Run

Consider the same production function:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

Suppose that input 2 is fixed in the short run at $\bar{x}_2 > 0$, while input 1 remains variable.

3.A. Write down the firm's short-run cost minimization problem.

3.B. Solve for the firm's optimal choice of x_1 as a function of $\bar{y}$ and $\bar{x}_2$.

3.C. Derive the short-run cost function $C^{SR}(w_1, \bar{x}_2, \bar{y})$.

3.D. Compare the short-run cost function to the long-run cost function from Problem 2. Which one is higher for a given level of output? Explain why.

Problem 3. Cost Minimization in the Short Run (continued)

3.E. Illustrate the firm's short-run cost minimization problem graphically. In your diagram, include:

- An isoquant corresponding to $\bar{y}$
- An isocost line
- The fixed level of input 2
- The firm's optimal choice of inputs

Explain how this differs from the long-run case.

3.F. True or False: "In the short run, the firm can never satisfy $MRTS_{12} = \frac{w_1}{w_2}$." Explain your answer.

3.G. Suppose the fixed input level $\bar{x}_2$ is very large. How does this affect the firm's short-run cost relative to the long run? Provide intuition.

Problem 4. Cost Curves

Suppose that a firm's total cost function is given by:

$$C(y) = 2y^2 + 6y + 20$$

4.A. Identify the firm's fixed cost, variable cost, and total cost.

4.B. Derive the firm's marginal cost (MC), average variable cost (AVC), and average total cost (ATC).

4.C. At what level of output is average total cost minimized? Show your work.

4.D. True or False: "If marginal cost is increasing, then average *variable* cost must also be increasing." Explain your answer.

Problem 4. Cost Curves (continued)

4.E. Graph the MC, AVC, and ATC curves. Clearly label:

- The minimum of the ATC curve
- The point where MC intersects ATC
- The relative positions of AVC and ATC

4.F. Explain why average fixed cost declines as output increases. What does this imply about the difference between ATC and AVC as output becomes large?

4.G. Explain why the AVC curve is typically upward sloping using the concept of diminishing marginal product.

4.H. Can the marginal cost curve ever lie below the average variable cost curve? Explain.

Problem 5. Firm Supply

Suppose that a competitive firm's total cost function is given by:

$$C(y) = y^2 + 2y + 16$$

5.A. Find the firm's marginal cost, average variable cost, and average total cost.

5.B. Find the output level that satisfies the firm's first-order condition for profit maximization at price p.

5.C. Determine the firm's short-run supply function.

5.D. Suppose that the market price is $p = 12$. Find the firm's optimal output and profit.

Problem 5. Firm Supply (continued)

5.E. Suppose that the market price is $p = 6$. Find the firm's optimal output and profit.

5.F. Suppose that the market price is $p = 2$. Find the firm's optimal output and profit.

5.G. Illustrate the firm's producer surplus at $p = 6$ graphically. Clearly label:

- The firm's marginal cost curve
- The market price
- The firm's chosen output
- The region corresponding to producer surplus

Problem 6. Industry Supply

Suppose that there are n identical competitive firms in a market. Each firm has the total cost function:

$$C(y) = y^2 + 2y + 16$$

6.A. Describe how the short-run industry supply curve is constructed from individual firm supply curves.

6.B. Suppose the number of firms increases from $n = 10$ to $n = 20$. What happens to industry supply? Explain graphically or intuitively.

6.C. Holding demand fixed, explain how an increase in the number of firms affects the equilibrium price and quantity.

6.D. Suppose that firms are earning positive profit in the short run. Describe what happens over time. How does this affect industry supply and market price?

Problem 6. Industry Supply (continued)

6.E. True or False: "In the long run, the market price is determined by demand." Explain your answer.

6.F. In the long run, what condition must hold for firms in equilibrium? In your answer, describe both the profit condition and the relationship between price and cost.

- Score: NOT GRADED
- Date: _____