CONCORDIA COLLEGE
MOORHEAD, MINNESOTA

- Name: ____________________
- Date: ____________________

BUSN 301: Intermediate Microeconomic Theory

Problem Set #3: Optional
Suggested Solutions

Spring 2026

INSTRUCTIONS:

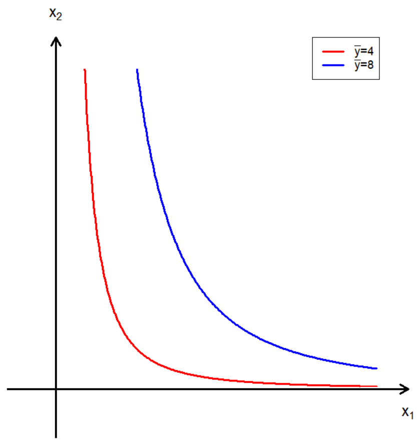
- This problem set is ungraded and is provided as a supplemental resource to the course.

Problem 1. The Production Function

Suppose that a producer's production technology is represented by the following production function:

$$f(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$$

1.A. Plot and compare the isoquants corresponding to $\bar{y} = 4$ and $\bar{y} = 8$.



- To find the equation of the isoquant for some level of output $\bar{y}$:

$$\bar{y} = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}} \quad \Rightarrow \quad x_2^{\frac{1}{3}} = \frac{\bar{y}}{x_1^{\frac{2}{3}}}$$

$$\Rightarrow \quad x_2 = \left(\frac{\bar{y}}{x_1^{\frac{2}{3}}} \right)^3$$

1.B. Find the expression for the marginal product of each factor of production.

$$MP_1 \equiv \frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}}$$

$$MP_2 \equiv \frac{\partial f(x_1, x_2)}{\partial x_2} = \frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}}$$

1.C. Which input exhibits stronger diminishing marginal product? Explain using your expressions.

- MP_1 decreases proportionally with $x_1^{-\frac{1}{3}}$.
- MP_2 decreases proportionally with $x_2^{-\frac{2}{3}}$.
- Since $-2/3 < -1/3$, MP_2 declines more rapidly as its input increases. Therefore, x_2 exhibits stronger diminishing marginal product.

1.D. Find the expression for the marginal rate of technical substitution between inputs 1 and 2.

$$MRTS_{1,2} \equiv \frac{MP_1}{MP_2} = \frac{\frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}}}{\frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}}} = \frac{2x_2}{x_1}$$

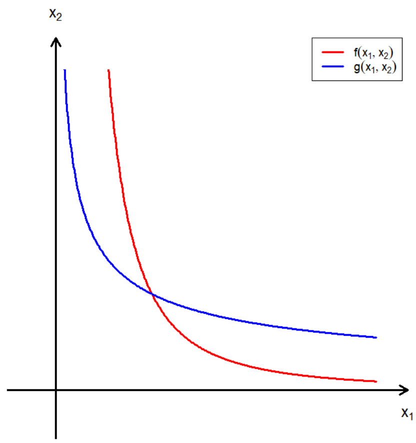
Problem 1. The Production Function (continued)

1.E. Determine whether the production function exhibits increasing, constant, or decreasing returns to scale. Show your work by evaluating $f(\lambda x_1, \lambda x_2)$ for $\lambda > 0$, and comparing it to $\lambda f(x_1, x_2)$.

- The production function exhibits constant returns to scale:

$$f(\lambda x_1, \lambda x_2) = (\lambda x_1)^{\frac{2}{3}}(\lambda x_2)^{\frac{1}{3}} = \lambda^{\frac{2}{3}} x_1^{\frac{2}{3}} \lambda^{\frac{1}{3}} x_2^{\frac{1}{3}} = \lambda x_1^{\frac{2}{3}} x_2^{\frac{1}{3}} = \lambda f(x_1, x_2)$$

1.F. Consider another production technology $g(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$. Plot the isoquants of f and g, each corresponding to $\bar{y} = 1$. (You may find it helpful to rewrite each isoquant as x_2 as a function of x_1.)



- To find the equation of the isoquant for g:

$$\bar{y} = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} \quad \Rightarrow \quad x_2^{\frac{2}{3}} = \frac{\bar{y}}{x_1^{\frac{1}{3}}}$$

$$\Rightarrow \quad x_2 = \left(\frac{\bar{y}}{x_1^{\frac{1}{3}}}\right)^{\frac{3}{2}}$$

1.G. Compare the two isoquants. Which input is used more intensively in each production function? Explain your answer.

- The production function $f(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$ uses input 1 more intensively, while $g(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$ uses input 2 more intensively.
- This can be seen from the exponents in each production function. In f, the exponent on x_1 is larger ($2/3 > 1/3$), indicating that output is more sensitive to changes in input 1. Therefore, the firm uses relatively more of input 1.
- Similarly, in g, the exponent on x_2 is larger, so the firm uses relatively more of input 2.
- Graphically, this is reflected in the shape of the isoquants. The isoquant for f is steeper, indicating a higher marginal rate of technical substitution of input 1 for input 2. In contrast, the isoquant for g is flatter, reflecting greater reliance on input 2.

Problem 2. Cost Minimization in the Long Run

Consider a firm with the following production function:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

The prices of inputs are given by ω_1 and ω_2, and the firm wishes to produce $\bar{y} > 0$ units of output.

2.A. Set up the firm's cost minimization problem.

$$\min_{x_1, x_2} \omega_1 x_1 + \omega_2 x_2 \quad s.t. \quad x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = \bar{y}$$

2.B. Derive the condition that characterizes the firm's optimal choice of inputs. Interpret this condition in economic terms.

$$MRTS_{1,2} = \frac{\omega_1}{\omega_2} \quad \Rightarrow \quad \frac{\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}}{\frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}} = \frac{\omega_1}{\omega_2} \quad \Rightarrow \quad \frac{x_2}{x_1} = \frac{\omega_1}{\omega_2} \quad \Rightarrow \quad x_2 = \frac{\omega_1}{\omega_2} x_1$$

- At the cost-minimizing bundle, the slope of the isoquant equals the slope of the isocost line. The firm chooses inputs so that the rate at which technology allows substitution between inputs equals the rate at which the market allows substitution through input prices.

2.C. Solve for the firm's conditional factor demands $x_1^*(\omega_1, \omega_2, \bar{y})$ and $x_2^*(\omega_1, \omega_2, \bar{y})$.

$$\bar{y} = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = x_1^{\frac{1}{2}} \left(\frac{\omega_1}{\omega_2} x_1\right)^{\frac{1}{2}} = x_1^{\frac{1}{2}} \left(\frac{\omega_1}{\omega_2}\right)^{\frac{1}{2}} x_1^{\frac{1}{2}} \quad \Rightarrow \quad \bar{y} = \left(\frac{\omega_1}{\omega_2}\right)^{\frac{1}{2}} x_1$$

$$\Rightarrow \quad x_1^*(\omega_1, \omega_2, \bar{y}) = \left(\frac{\omega_2}{\omega_1}\right)^{\frac{1}{2}} \bar{y}$$

$$\Rightarrow \quad x_2^*(\omega_1, \omega_2, \bar{y}) = \left(\frac{\omega_1}{\omega_2}\right)^{\frac{1}{2}} \bar{y}$$

2.D. Derive the firm's cost function $C(\omega_1, \omega_2, \bar{y})$.

$$C(\omega_1, \omega_2, \bar{y}) = \omega_1 x_1^* + \omega_2 x_2^* = \omega_1 \left(\frac{\omega_2}{\omega_1}\right)^{\frac{1}{2}} \bar{y} + \omega_2 \left(\frac{\omega_1}{\omega_2}\right)^{\frac{1}{2}} \bar{y} \quad \Rightarrow \quad C(\omega_1, \omega_2, \bar{y}) = 2\left(\omega_1 \omega_2\right)^{\frac{1}{2}} \bar{y}$$

Problem 2. Cost Minimization in the Long Run (continued)

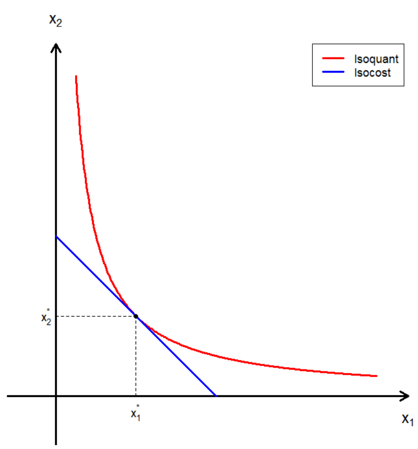
2.E. Suppose that $\omega_1 = \omega_2$. What does this imply about the firm's optimal choice of inputs? Explain your answer.

- When $\omega_1 = \omega_2$, the firm's optimal choice of inputs satisfy $\frac{x_2}{x_1} = \frac{\omega_1}{\omega_2} = 1$.
- This means that the firm chooses the two inputs in equal amounts.
- Intuitively, when the two inputs have the same price, the firm has no cost-based reason to favor one input over the other.
- Since the production function is symmetric in x_1 and x_2, the cost-minimizing bundle uses equal quantities of both inputs.

2.F. Illustrate the firm's cost minimization problem graphically. In your diagram, include:

- An isoquant corresponding to $\bar{y}$
- An isocost line
- The optimal bundle of inputs

Explain how the condition from part 2.B is reflected in your graph.



- The optimal bundle occurs at the tangency point between the isoquant and the isocost line.
- At this point, the slope of the isoquant equals the slope of the isocost line. Therefore:

$$MRTS_{1,2} = \frac{\omega_1}{\omega_2}$$

Problem 3. Cost Minimization in the Short Run

Consider the same production function:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

Suppose that input 2 is fixed in the short run at $\bar{x}_2 > 0$, while input 1 remains variable.

3.A. Write down the firm's short-run cost minimization problem.

$$\min_{x_1} \omega_1 x_1 + \omega_2 \bar{x}_2 \quad s.t. \quad x_1^{\frac{1}{2}} \bar{x}_2^{\frac{1}{2}} = \bar{y}$$

3.B. Solve for the firm's optimal choice of x_1 as a function of $\bar{y}$ and $\bar{x}_2$.

$$x_1^{\frac{1}{2}} \bar{x}_2^{\frac{1}{2}} = \bar{y} \quad \Rightarrow \quad x_1^{\frac{1}{2}} = \bar{y} \bar{x}_2^{-\frac{1}{2}} \quad \Rightarrow \quad x_1^*(\bar{y}, \bar{x}_2) = \frac{\bar{y}^2}{\bar{x}_2}$$

3.C. Derive the short-run cost function $C^{SR}(w_1, \bar{x}_2, \bar{y})$.

$$C^{SR}(\omega_1, \bar{x}_2, \bar{y}) = \omega_1 x_1^* + \omega_2 \bar{x}_2 = \omega_1 \frac{\bar{y}^2}{\bar{x}_2} + \omega_2 \bar{x}_2$$

3.D. Compare the short-run cost function to the long-run cost function from Problem 2. Which one is higher for a given level of output? Explain why.

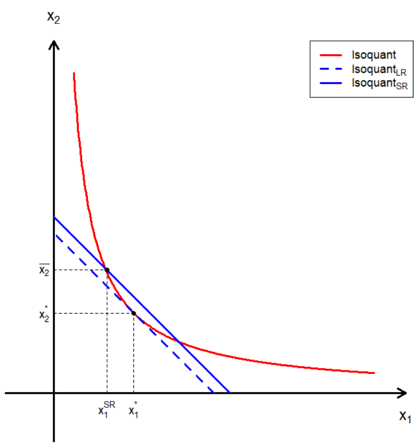
- The short-run cost function is: $C^{SR}(w_1, \bar{x}_2, \bar{y}) = w_1 \frac{\bar{y}^2}{\bar{x}_2} + w_2 \bar{x}_2$
- The long-run cost function from Problem 2 is: $C(w_1, w_2, \bar{y}) = 2(w_1 w_2)^{1/2} \bar{y}$
- For a given level of output, the short-run cost is greater than or equal to the long-run cost: $C^{SR}(w_1, \bar{x}_2, \bar{y}) \geq C(w_1, w_2, \bar{y})$
- This is because in the long run the firm can adjust both inputs freely, while in the short run input 2 is fixed at $\bar{x}_2$. Since the firm has fewer choices in the short run, it cannot do better than the long-run minimum cost. At best, the short-run cost equals the long-run cost if the fixed input level happens to coincide with the long-run cost-minimizing choice.

Problem 3. Cost Minimization in the Short Run (continued)

3.E. Illustrate the firm's short-run cost minimization problem graphically. In your diagram, include:

- An isoquant corresponding to $\bar{y}$
- An isocost line
- The fixed level of input 2
- The firm's optimal choice of inputs

Explain how this differs from the long-run case.



- In the short run, input 2 is fixed at $\bar{x}_2$. Graphically, this means the firm's choice must lie on the vertical line at $x_2 = \bar{x}_2$. The firm chooses the point where this vertical line intersects the isoquant corresponding to $\bar{y}$. This gives the cost-minimizing level of input 1, given that input 2 is fixed.
- In the long run, the firm can adjust both inputs freely and chooses the tangency point between the isoquant and the isocost line. In the short run, that tangency generally does not occur, because the firm is constrained by the fixed level of input 2.

3.F. True or False: "In the short run, the firm can never satisfy $MRTS_{12} = \frac{w_1}{w_2}$." Explain your answer.

- False.
- In general, the firm may not be able to satisfy $MRTS_{12} = \frac{w_1}{w_2}$ in the short run.
- However, if the fixed quantity of input $\bar{x}_2$ happens to coincide with the long run cost minimizing x_2^* for some given target output $\bar{y}$, $MRTS_{12} = \frac{w_1}{w_2}$ may hold.

3.G. Suppose the fixed input level $\bar{x}_2$ is very large. How does this affect the firm's short-run cost relative to the long run? Provide intuition.

- If the fixed input level $\bar{x}_2$ is very large, the firm's short-run cost will generally be higher than its long-run cost.
- The reason is that, in the short run, the firm is stuck with this large amount of input 2 and cannot reduce it, even if it is more than the cost-minimizing amount. As a result, the firm may use an inefficient input combination relative to the long run.
- In the long run, the firm would adjust both inputs freely and choose the least-cost bundle. In the short run, the excess amount of input 2 acts like a constraint, so the firm cannot fully optimize. Therefore, short-run cost remains weakly above long-run cost.

Problem 4. Cost Curves

Suppose that a firm's total cost function is given by:

$$C(y) = 2y^2 + 6y + 20$$

4.A. Identify the firm's fixed cost, variable cost, and total cost.

$$FC = 20$$
$$VC = 2y^2 + 6y$$
$$TC = 2y^2 + 6y + 20$$

4.B. Derive the firm's marginal cost (MC), average variable cost (AVC), and average total cost (ATC).

$$AVC(y) \equiv \frac{VC}{y} = 2y + 6$$
$$ATC(y) \equiv \frac{TC}{y} = 2y + 6 + \frac{20}{y}$$
$$MC(y) \equiv \frac{dTC}{dy} = 4y + 6$$

4.C. At what level of output is average total cost minimized? Show your work.

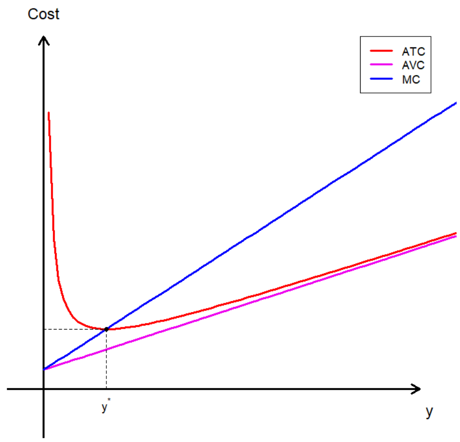
$$\frac{dATC}{dy} = 0 \quad \Rightarrow \quad 2 - \frac{20}{y^2} = 0 \quad \Rightarrow \quad y = \sqrt{10}$$

4.D. True or False: "If marginal cost is increasing, then average *variable* cost must also be increasing." Explain your answer.

- An increasing marginal cost curve does not necessarily imply that average variable cost is increasing at that output level. What matters for whether AVC is rising or falling is the relationship between MC and AVC:
  - If $MC > AVC$, then AVC is increasing.
  - If $MC < AVC$, then AVC is decreasing.
- So even if marginal cost is increasing, average variable cost could still be decreasing, as long as MC remains below AVC.

Problem 4. Cost Curves (continued)

4.E. Graph the MC, AVC, and ATC curves:



- The MC curve intersects both the AVC and ATC curves at their minimum points.
- At y^*, the MC curve intersects the ATC curve at its minimum.
- For output levels where $MC < ATC$, ATC is decreasing. For output levels where $MC > ATC$, ATC is increasing.

4.F. Explain why average fixed cost declines as output increases. What does this imply about the difference between ATC and AVC as output becomes large?

- Average fixed cost is given by $\frac{FC}{y}$. Since fixed cost does not change with output, increasing y spreads the same fixed cost over more units of output. As a result, as $y \to \infty$, $AFC \to 0$.
- Since $ATC = AVC + AFC$, this implies that the difference between ATC and AVC becomes smaller as output increases. In the limit, ATC approaches AVC.

4.G. Explain why the AVC curve is typically upward sloping using the concept of diminishing marginal product.

- The upward-sloping shape of the AVC curve is driven by diminishing marginal product.
- In the short run, some inputs are fixed. As the firm increases output by using more of the variable input, the MP of that input eventually declines due to diminishing returns.
- Since MP falls, the firm needs increasingly more units of the variable input to produce each additional unit of output. This raises the cost of producing each unit.
- As a result, variable cost per unit of output increases, and the AVC curve is upward sloping.

4.H. Can the marginal cost curve ever lie below the average variable cost curve? Explain.

- Yes.
- The MC curve lies below the AVC curve when AVC is decreasing. In this case, $MC < AVC$, and each additional unit of output lowers the AVC.
- Once MC rises above AVC, AVC begins to increase. Therefore, the MC curve intersects the AVC curve at the minimum point of AVC.

Problem 5. Firm Supply

Suppose that a competitive firm's total cost function is given by:

$$C(y) = y^2 + 2y + 16$$

5.A. Find the firm's marginal cost, average variable cost, and average total cost.

$$AVC(y) \equiv \frac{VC}{y} = \frac{y^2 + 2y}{y} = y + 2$$
$$ATC(y) \equiv \frac{TC}{y} = \frac{y^2 + 2y + 16}{y} = y + 2 + \frac{16}{y}$$
$$MC(y) \equiv \frac{dTC}{dy} = 2y + 2$$

5.B. Find the output level that satisfies the firm's first-order condition for profit maximization at price p.

$$\frac{d\pi}{dy} = 0 \quad \Rightarrow \quad \frac{d}{dy} p \cdot y - C(y) = 0 \quad \Rightarrow \quad p - MC(y) = 0 \quad \Rightarrow \quad p - 2y - 2 = 0$$
$$\Rightarrow \quad y^* = \frac{p-2}{2}$$

5.C. Determine the firm's short-run supply function.

- The supply curve is the MC that is increasing in y, which is true for all values of y.
- The supply curve is the portion of the MC curve that lies above the AVC curve.

$$MC \geq AVC \quad \Rightarrow \quad 2y + 2 \geq y + 2 \quad \Rightarrow \quad y \geq 0$$

- So, the firm's supply function is:

$$S(p) = \begin{cases} \frac{p-2}{2}, & p \geq 2 \\ 0, & \text{otherwise} \end{cases}$$

5.D. Suppose that the market price is $p = 12$. Find the firm's optimal output and profit.

- According to the supply function, the firm's optimal output is 5 units.
- The firm's profit is then:

$$\pi = 12 \cdot 5 - \left(5^2 + 2 \cdot 5 + 16\right) = 9$$

Problem 5. Firm Supply (continued)

5.E. Suppose that the market price is $p = 6$. Find the firm's optimal output and profit.

- According to the supply function, the firm's optimal output is 2 units.
- The firm's profit is then:

$$\pi = 6 \cdot 2 - \left(2^2 + 2 \cdot 2 + 16\right) = -12$$

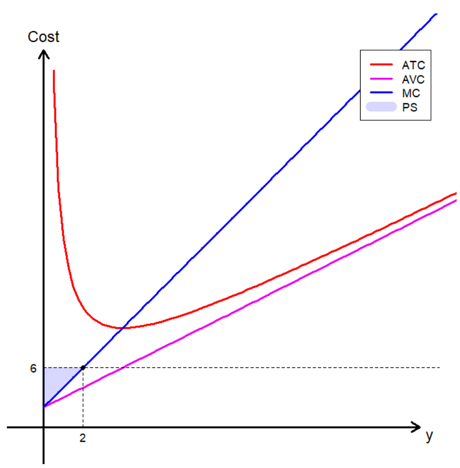
5.F. Suppose that the market price is $p = 2$. Find the firm's optimal output and profit.

- $p = 2$ is the firm's shutdown price, so the firm's optimal output is 0 units.
- The firm's profit is then:

$$\pi = 2 \cdot 0 - \left(0^2 + 2 \cdot 0 + 16\right) = -16$$

5.G. Illustrate the firm's producer surplus at $p = 6$ graphically. Clearly label:

- The firm's marginal cost curve
- The market price
- The firm's chosen output
- The region corresponding to producer surplus



- At $p = 6$, the firm chooses $y^* = 2$.
- Producer surplus is the area above (or, to the left of) the marginal cost curve, and below the market price, from $y = 0$ to $y^* = 2$.
- This area represents the difference between what the firm receives for each unit sold and the marginal cost of producing that unit.

Problem 6. Industry Supply

Suppose that there are n identical competitive firms in a market. Each firm has the total cost function:

$$C(y) = y^2 + 2y + 16$$

6.A. Describe how the short-run industry supply curve is constructed from individual firm supply curves.

- The short-run industry supply curve is the horizontal sum of all individual firms' supply curves.

6.B. Suppose the number of firms increases from $n = 10$ to $n = 20$. What happens to industry supply? Explain graphically or intuitively.

- For any price above the shutdown price, industry supply doubles when the number of firms increases from $n = 10$ to $n = 20$.

6.C. Holding demand fixed, explain how an increase in the number of firms affects the equilibrium price and quantity.

- Holding demand fixed, an increase in the number of firms shifts industry supply to the right. As a result, equilibrium price falls and equilibrium quantity rises.

6.D. Suppose that firms are earning positive profit in the short run. Describe what happens over time. How does this affect industry supply and market price?

- Positive short-run profits induce entry by new firms, which shifts industry supply to the right and pushes market price downward.
- As long as firms continue to earn positive economic profit, entry continues.
- Entry stops when the market price reaches $\min ATC$, so that firms earn zero economic profit in long-run equilibrium.

Problem 6. Industry Supply (continued)

6.E. True or False: "In the long run, the market price is determined by demand." Explain your answer.

- False.
- In the long run, the market price is determined by firms' cost conditions. In equilibrium, $p^* = \min ATC$.
- Demand still plays a role by determining the total quantity produced and the number of firms in the market, but it does not determine the equilibrium price.

6.F. In the long run, what condition must hold for firms in equilibrium? In your answer, describe both the profit condition and the relationship between price and cost.

- In the long-run equilibrium, firms earn zero economic profit.
- This implies that price equals average total cost:$p^* = ATC$.
- In equilibrium, firms operate at the minimum of the ATC curve, so $p^* = \min ATC$.
- This outcome arises due to free entry and exit: positive profits induce entry, while losses induce exit, driving profits to zero.

- Score: `NOT GRADED`
- Date: ____________________