- Name: ______________________________

- Date: ______________________________

BUSN 301: Intermediate Microeconomic Theory

Problem Set #2: Suggested Solutions

Spring 2026

INSTRUCTIONS:

- Each problem set is graded on a 100-point basis and contributes to your Problem Set component of the course grade.
- You are expected to show all relevant steps and reasoning.
- Answers must be clearly written and well-organized.
- Graphs, when required, must be clearly labeled, with axes, curves, and key points identified.
- Problem sets must be submitted by the posted deadline.

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Problem 1. Walrasian Demand

Suppose that a consumer's utility function is given as $u(x_1, x_2) = 2x_1^{\frac{1}{3}}x_2^{\frac{2}{3}}$. The unit prices of good 1 and good 2 are denoted by p_1 and p_2, respectively. The consumer has an income of m.

1.A. Compute the marginal utility with respect to good 1 and good 2.

$$MU_1 = \frac{\partial}{\partial x_1}2x_1^{\frac{1}{3}}x_2^{\frac{2}{3}} = \frac{2}{3}x_1^{-\frac{2}{3}}x_2^{\frac{2}{3}}$$
$$MU_2 = \frac{\partial}{\partial x_2}2x_1^{\frac{1}{3}}x_2^{\frac{2}{3}} = \frac{4}{3}x_1^{\frac{1}{3}}x_2^{-\frac{1}{3}}$$

1.B. Write down the consumer's utility maximization problem and derive the first-order (tangency) condition.

$$MRS = \frac{p_1}{p_2} \quad \Rightarrow \quad \frac{\frac{2}{3}x_1^{-\frac{2}{3}}x_2^{\frac{2}{3}}}{\frac{4}{3}x_1^{\frac{1}{3}}x_2^{-\frac{1}{3}}} = \frac{p_1}{p_2}$$
$$\Rightarrow \quad \frac{x_2}{2x_1} = \frac{p_1}{p_2}$$
$$\Rightarrow \quad x_2 = \frac{2p_1}{p_2}x_1$$

1.C. Write down the equation describing the consumer's budget line.

$$p_1x_1 + p_2x_2 = m$$

1.D. Derive the consumer's Walrasian demand function for good 1, $x_1(p_1, p_2, m)$.

$$p_1x_1 + p_2x_2 = m \quad \Rightarrow \quad p_1x_1 + p_2\left(\frac{2p_1}{p_2}x_1\right) = m$$
$$\Rightarrow \quad p_1x_1 + 2p_1x_1 = m$$
$$\Rightarrow \quad 3p_1x_1 = m$$
$$\Rightarrow \quad x_1 = \frac{1}{3}\frac{m}{p_1}$$

Problem 1. Walrasian Demand (continued)

1.E. Calculate the price elasticity of demand for good 1.

$$\varepsilon = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1} = \left(-\frac{m}{3p_1^2}\right)\frac{p_1}{\frac{m}{3p_1}} = -1$$

1.F. Based on your answer to 1.E, is good 1 an ordinary good or a Giffen good? Briefly explain.

- Since the price elasticity of demand is -1, we know that the quantity demanded of good 1 decreases following an increase in prices. Therefore, good 1 is an ordinary good.

1.G. If the price of good 1 increased by 5%, what happens to total revenue earned by sellers of good 1? Briefly explain.

- Since the price elasticity of demand is -1, we know that the quantity demanded of good 1 decreases by 5% following a 5% increase in prices. Therefore, revenue remains unchanged.
- *Caveat: 5% may be outside of the bounds of marginal approximation, making real-world application of this result a bit tricky.*

1.H. Calculate the income elasticity of demand for good 1.

$$\varepsilon = \frac{\partial x_1}{\partial m}\frac{m}{x_1} = \left(\frac{1}{3p_1}\right)\frac{m}{\frac{m}{3p_1}} = 1$$

Problem 2. The Engel Curve

Suppose that a consumer's utility function is given as $u(x_1, x_2) = (0.4x_1^2 + 0.6x_2^2)^{\frac{1}{2}}$. This utility function is known as a Constant Elasticity of Substitution (CES) utility function. The unit prices of goods 1 and 2 are denoted by p_1 and p_2, respectively. The consumer has an income of m. The marginal rate of substitution between goods 1 and 2 is given as follows:

$$MRS = \frac{MU_1}{MU_2} = \frac{2x_1}{3x_2}$$

2.A. *OPTIONAL:* Compute the marginal utility with respect to good 1 and good 2.
Hint: Use the chain rule: $\frac{d}{dx} f(g(x)) = g'(x) f'(g(x))$.

$$MU_1 = \frac{\partial}{\partial x_1} \left(0.4x_1^2 + 0.6x_2^2\right)^{\frac{1}{2}} = 0.4x_1 \left(0.4x_1^2 + 0.6x_2^2\right)^{-\frac{1}{2}}$$
$$MU_2 = \frac{\partial}{\partial x_2} \left(0.4x_1^2 + 0.6x_2^2\right)^{\frac{1}{2}} = 0.6x_2 \left(0.4x_1^2 + 0.6x_2^2\right)^{-\frac{1}{2}}$$

2.B. Write down the consumer's utility maximization problem and derive the first-order (tangency) condition.

$$\frac{MU_1}{MU_2} = \frac{p_1}{p_2} \quad \Rightarrow \quad \frac{2x_1}{3x_2} = \frac{p_1}{p_2} \quad \Rightarrow \quad x_2 = \frac{2p_2}{3p_1} x_1$$

2.C. Write down the equation describing the consumer's budget line.

$$p_1 x_1 + p_2 x_2 = m$$

2.D. Derive the expression for the consumer's Engel curve for good 1, $x_1(m, p_1, p_2)$.

$$\begin{aligned} p_1 x_1 + p_2 x_2 = m \quad &\Rightarrow \quad p_1 x_1 + p_2 \left(\frac{2p_2}{3p_1} x_1\right) = m \\ &\Rightarrow \quad \left(p_1 + \frac{2p_2^2}{3p_1}\right) x_1 = m \\ &\Rightarrow \quad \left(\frac{3p_1^2}{3p_1} + \frac{2p_2^2}{3p_1}\right) x_1 = m \\ &\Rightarrow \quad x_1 = \left(\frac{3p_1}{3p_1^2 + 2p_2^2}\right) m \end{aligned}$$

Problem 2. The Engel Curve (continued)

2.E. Compute the income elasticity of demand for good 1.

$$\varepsilon = \frac{\partial x_1}{\partial m}\frac{m}{x_1} = \left(\frac{3p_1}{3p_1^2 + 2p_2^2}\right)\frac{m}{\left(\frac{3p_1}{3p_1^2+2p_2^2}\right)m} = 1$$

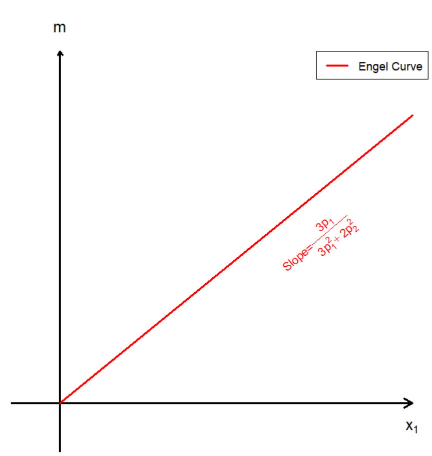
2.F. Based on your answer to 2.E, is good 1 a normal good or inferior good? Briefly explain.

- Since the income elasticity of demand is 1, we know that the quantity demanded of good 1 increases following an increase in income. Therefore, good 1 is a normal good.

2.G. Is the Engel curve linear, convex, or concave in income? Briefly explain.

- Since prices are fixed and given, the curve is linear.

2.H. Graph the Engel curve for good 1. Clearly label axes and indicate whether the good is normal or inferior.



Problem 3. Individual and Market Demand

Suppose there are three consumers in a market for a single good. Assume each consumer's demand cannot be negative (that is, $x_i(p) = 0$ whenever the expression above is negative). Each consumer has an individual demand function given by:

$$\begin{cases} x_A = 20 - 2p \\ x_B = 20 - p \\ x_C = 20 - \frac{1}{2}p \end{cases}$$

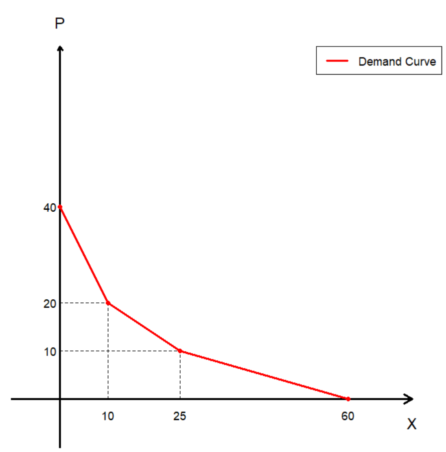
3.A. For each consumer, compute the choke price (the price at which quantity demanded becomes zero).

$$\begin{cases} 0 = 20 - 2p & \Rightarrow \quad p_A = 10 \\ 0 = 20 - p & \Rightarrow \quad p_B = 20 \\ 0 = 20 - \frac{1}{2}p & \Rightarrow \quad p_C = 40 \end{cases}$$

3.B. Derive the market demand function $X(p)$ by horizontally summing the individual demand functions. Your final answer should be written as a piecewise function.

$$X(p) = \begin{cases} 60 - \frac{7}{2}p, & 0 \leq p \leq 10 & \because x_A + x_B + x_C \\ 40 - \frac{3}{2}p, & 10 < p \leq 20 & \because x_B + x_C \\ 20 - \frac{1}{2}p, & 20 < p \leq 40 & \because x_C \\ 0, & p > 40 \end{cases}$$

3.C. Graph the market demand curve. Clearly label all intercepts and any kink points.



Problem 4. Equilibrium

Suppose the market demand (D) and supply (S) functions in a competitive market are given by:

$$\begin{cases} D(p) = 200 - 2p \\ S(p) = 3p \end{cases}$$

4.A. Determine the equilibrium price p^* and quantity traded in the market q^*.

$$D(p^*) = S(p^*) \quad \Rightarrow \quad 200 - 2p^* = 3p^* \quad \Rightarrow \quad 5p^* = 200 \quad \Rightarrow \quad p^* = 40$$

Plugging in the equilibrium price in either the demand or supply function, we can find the equilibrium quantity:

$$q^* = D(40) = 200 - 2 \cdot 40 = 120$$

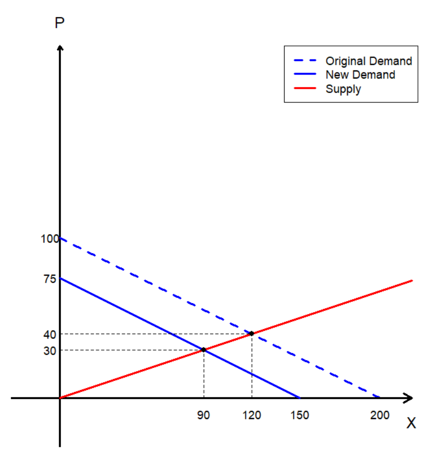
4.B. Demand is now $D'(p) = 150 - 2p$. Determine the new equilibrium price and quantity.

$$D'(p') = S(p') \quad \Rightarrow \quad 150 - 2p' = 3p' \quad \Rightarrow \quad 5p' = 150 \quad \Rightarrow \quad p' = 30$$

Plugging in the equilibrium price in either the demand or supply function, we can find the equilibrium quantity:

$$q' = D'(30) = 150 - 2 \cdot 30 = 90$$

4.C. In a single graph, plot the original demand (D), new demand (D'), and supply curve. Clearly label all intercepts and both market equilibria.



Problem 4. Equilibrium (continued)

4.D. Calculate the original consumer surplus and producer surplus under the original supply and demand curves.

- $CS : 120 \cdot (100 - 40) \cdot \frac{1}{2} = 3,600$
- $PS : 120 \cdot 40 \cdot \frac{1}{2} = 2,400$

4.E. Calculate the new consumer surplus and producer surplus under the new demand curve.

- $CS' : 90 \cdot (75 - 30) \cdot \frac{1}{2} = 2,025$
- $PS' : 90 \cdot 30 \cdot \frac{1}{2} = 1,350$

4.F. Calculate total surplus before and after the demand shift. By how much does total surplus change?

- $TS : CS + PS = 3,600 + 2,400 = 6,000$
- $TS' : CS' + PS' = 2,025 + 1,350 = 3,375$
- The total surplus decreased by 2,625.

4.G. Compute the price elasticity of demand and price elasticity of supply at the original equilibrium. Which side of the market is more elastic?

- $\varepsilon_D = \frac{dQ}{dp} \cdot \frac{p^*}{Q^*} = -2 \cdot \frac{40}{120} = -\frac{2}{3}$
- $\varepsilon_S = \frac{dQ}{dp} \cdot \frac{p^*}{Q^*} = 3 \cdot \frac{40}{120} = 1$
- Supply is more elastic than demand at the original equilibrium.

Problem 5. Taxation

Suppose the market demand (D) and supply (S) functions in a competitive market are given by:

$$\begin{cases} D(p) = 200 - 2p \\ S(p) = 3p \end{cases}$$

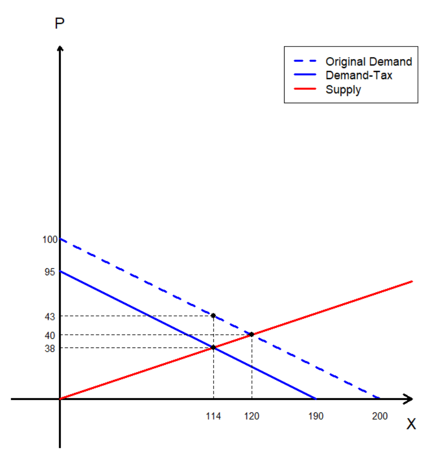
5.A. Suppose that a per-unit tax of $t = 5$ is levied. Determine the price to consumers p_d, price to producers p_s, and the quantity traded q_t in this market.

$$D(p_d) = S(p_s) \quad \Rightarrow \quad 200 - 2p_d = 3p_s \quad \Rightarrow \quad 200 - 2(p_s + 5) = 3p_s \quad \Rightarrow \quad p_s = 38$$

Using the relationship $p_d = p_s + t$, and plugging in the producer's price in the supply function:

$$p_d = p_s + 5 = 43$$
$$q_t = S(38) = 3 \cdot 38 = 114$$

5.B. In a single graph, plot the pre-tax and post-tax equilibria alongside the demand and supply curves. Clearly label p^*, q^*, p_d, p_s, and q_t.



5.C. Calculate total surplus in the pre-tax market.

$$TS = CS + PS = 6,000$$

5.D. Calculate total surplus in the post-tax market.

$$TS_t = CS_t + PS_t + \text{Tax Revenue} = \left\{114 \cdot (100 - 43) \cdot \frac{1}{2}\right\} + \left\{114 \cdot 38 \cdot \frac{1}{2}\right\} + \{114 \cdot 5\} = 5,985$$

Problem 5. Taxation (continued)

5.E. Calculate the per-unit and total tax burden for consumers and producers.

- Per-unit, the consumer's price increased by 3, while the producer's price decreased by 2.
- The consumer's share of the tax is $3 \cdot 114 = 342$.
- The producer's share of the tax is $2 \cdot 114 = 228$.
- The consumer's burden is $\frac{342}{342+228} = 0.6$.
- The producer's burden is $\frac{228}{342+228} = 0.4$.

5.F. Calculate the deadweight loss of taxation.

$$DWL = TS - TS_t = 6,000 - 5,985 = 15$$

5.G. Suppose that instead of the per-unit tax of $t = 5$, an ad valorem tax of $\tau = 10\%$ is levied. Determine the price to consumers p_d, price to producers p_s, and the quantity traded q_τ in this market.

$$D(p_d) = S(p_s) \quad \Rightarrow \quad 200 - 2p_d = 3p_s \quad \Rightarrow \quad 200 - 2(1.1 \cdot p_s) = 3p_s \quad \Rightarrow \quad p_s = \frac{500}{13} \simeq 38.46$$

Using the relationship $p_d = (1+\tau) \cdot p_s$, and plugging in the producer's price in the supply function, or the consumer's price in the demand function:

$$p_d = 1.1 \cdot p_s = \frac{550}{13} \simeq 42.31$$
$$q_\tau = S\left(\frac{500}{13}\right) = 3 \cdot \frac{500}{13} = \frac{1,500}{13} \simeq 115.38$$

- Score: ____________________
- Date: ____________________