CONCORDIA COLLEGE
MOORHEAD, MINNESOTA

- Name: ____________________________
- Date: ____________________________

BUSN 301: Intermediate Microeconomic Theory

Problem Set #1: Suggested Solutions

Spring 2026

INSTRUCTIONS:

- Each problem set is graded on a 100-point basis and contributes to your Problem Set component of the course grade.
- You are expected to show all relevant steps and reasoning.
- Answers must be clearly written and well-organized.
- Graphs, when required, must be clearly labeled, with axes, curves, and key points identified.
- Problem sets must be submitted by the posted deadline.

Problem 1. Budget Constraint: Basics

Suppose that the prices of good 1 (x_1) and good 2 (x_2) are given as $p_1 = 5$ and $p_2 = 4$, respectively. The consumer's income is $m = 60$, and the consumer allocates income across two goods only.

1.A. If the consumer spends their entire budget on only one good, determine the intercepts of the budget constraint. That is, calculate the maximum amount of x_1 that can be consumed when $x_2 = 0$, and the maximum amount of x_2 that can be consumed when $x_1 = 0$.

- The x_1-intercept is $m/p_1 = 60/5 = 12$.
- The x_2-intercept is $m/p_2 = 60/4 = 15$.

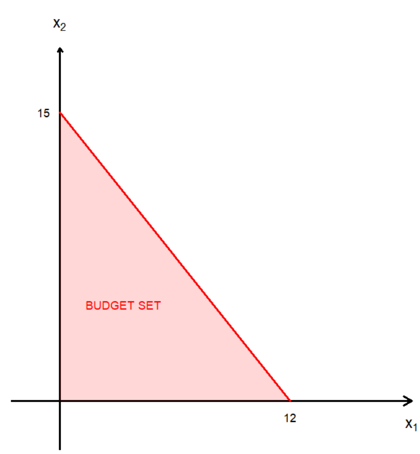
1.B. Is the bundle $(5, 7)$ in the consumer's budget set? Show total expenditure and compare it to income.

- Total expenditure is $p_1 \cdot 5 + p_2 \cdot 7 = 25 + 28 = 53 < 60 = m$.
- Since total expenditure is less than income, $(5, 7)$ lies in the consumer's budget set.

1.C. Derive the equation for the budget line. Then calculate the slope of the budget line and provide an economic interpretation in terms of trade-offs between x_1 and x_2.

- Budget line: $p_1x_1 + p_2x_2 = m$, or $5x_1 + 4x_2 = 60$.
- Slope: $-p_1/p_2 = -5/4$ (with x_1 on the horizontal axis).
- The slope represents the rate at which the consumer can trade good 1 for good 2 in the market, given prices.

1.D. Graph the consumer's budget constraint with x_1 on the horizontal axis and x_2 on the vertical axis. Clearly label the intercepts and indicate the feasible set.



Problem 2. Budget Constraint: Comparative Statics

Suppose the consumer faces the same initial prices and income as in Problem 1, with $p_1 = 5$, $p_2 = 4$, and $m = 60$. Now suppose that the price of good 1 increases to $p_1 = 10$, while income and the price of good 2 remain unchanged.

2.A. Write down the consumer's new budget set and derive the equation for the new budget line.

- The budget set is characterized by $p_1 x_1 + p_2 x_2 \leq m$.
- Substituting the given parameters, the budget line is $10x_1 + 4x_2 = 60$.

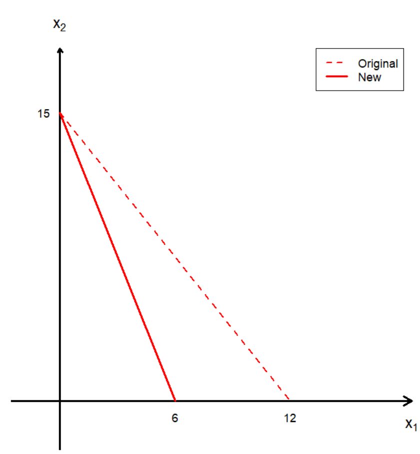
2.B. How do the intercepts of the budget constraint change as a result of the increase in p_1?

- The x_1-intercept is now $m/p_1 = 60/10 = 6$.
- The x_2-intercept remains unchanged at $m/p_2 = 60/4 = 15$.

2.C. Compare the slope of the new budget line to the slope of the original budget line. Has the budget constraint shifted or rotated?

- The slope becomes steeper in absolute value, changing from $-5/4$ to $-10/4$.
- The budget line pivots inward around the x_2-intercept, causing the budget set to contract.

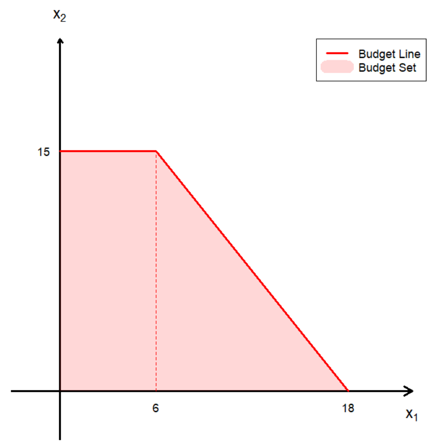
2.D. On a single graph, draw both the original and the new budget constraints. Clearly label all intercepts and indicate the direction of the change.



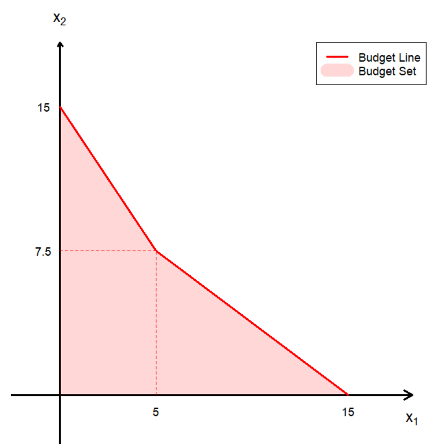
Problem 3. Budget Constraints: Non-Standard Budget Constraints

Suppose the consumer faces the same initial prices and income as in Problem 1, with $p_1 = 5$, $p_2 = 4$, and $m = 60$.

3.A. Suppose the first 6 units of good 1 are free, but any additional units of good 1 cost $p_1 = 5$ per unit. Graph the budget constraint.



3.B. Suppose instead that the consumer faces a quantity discount for good 1, where the first 5 units cost $p_1 = 6$ per unit, and any additional units cost $p_1 = 3$ per unit. Graph the budget constraint.



Problem 4. Preferences

Assume that the preference relation ($\succsim$) is well-behaved, as defined in class.

4.A. Suppose the consumer is indifferent between bundles X and Y, and indifferent between bundles Y and Z. What must be true about the relationship between X and Z? Which axiom of preference justifies your answer?

- By transitivity, $X \sim Y$ and $Y \sim Z$ implies $X \sim Z$.

4.B. Define an indifference curve. Explain why two indifference curves for the same preference relation cannot intersect.

- An indifference curve is the set of bundles that are indifferent to a given bundle.
- Two indifference curves cannot intersect because if they did, one bundle would be indifferent to two different bundles on different curves. By transitivity, those bundles would have to be indifferent to each other, contradicting that they lie on different indifference curves.

4.C. In your own words, define the axiom of convexity. What does convexity imply about the shape of the consumer's indifference curve?

- Convexity: Consumers prefer more balanced bundles to extreme bundles.
- Convex preferences imply indifference curves are bowed toward the origin.

4.D. In your own words, define the axiom of monotonicity. What does monotonicity imply about the shape of the consumer's indifference curve?

- Monotonicity: The more the better.
- Preference relations that are monotone cannot have an indifference curve that slopes upward.

4.E. Suppose bundle X lies on a higher indifference curve than bundle Y. Which bundle does the consumer prefer and why?

- The consumer prefers bundle X because higher indifference curves represent more preferred bundles.

Problem 5. Utility: Basics

Assume that the preference relation ($\succsim$) is well-behaved, as defined in class, and that $u(x_1, x_2)$ is a representation of this preference relation.

5.A. What does it mean for a utility function to be an ordinal representation of preferences?

- A utility function is an ordinal representation of preferences if it preserves the ranking of bundles.
- The numerical values themselves have no intrinsic meaning beyond ordering.

5.B. Why do monotonic transformations represent the same preferences?

- Any strictly increasing (monotonic) transformation preserves the rank ordering of bundles, so it represents the same preferences.

5.C. Is it possible for two different utility levels to correspond to the same indifference curve? Explain.

- No.
- An indifference curve consists of all bundles that yield the same utility level, and bundles with different utility values cannot lie on the same indifference curve.

5.D. In general, what does it mean for an indifference curve to be "further from the origin"? Why does this matter for ranking bundles?

- With monotone preferences, an indifference curve further from the origin represents bundles with more of at least one good and no less of the other, and is therefore preferred to an indifference curve closer to the origin.

5.E. Suppose bundle X lies on a higher indifference curve than bundle Y. What can we conclude about $u(X)$ and $u(Y)$? What does this say about the consumer's preferences?

- Since X lies on a higher indifference curve than Y, $u(X) > u(Y)$, and the consumer strictly prefers X to Y.

Problem 6. Utility Functions, Marginal Utility, and the Marginal Rate of Substitution

Suppose a consumer's utility function is given as $u(x_1, x_2) = 4x_1x_2^2$.

6.A. Compute the marginal utility of good 1 and the marginal utility of good 2.

- $MU_1 = \frac{\partial}{\partial x_1} 4x_1x_2^2 = 4x_2^2$
- $MU_2 = \frac{\partial}{\partial x_2} 4x_1x_2^2 = 8x_1x_2$

6.B. Using your answers from part `6.A`, derive the marginal rate of substitution (MRS).

$$MRS \equiv -\frac{MU_1}{MU_2} = -\frac{4x_2^2}{8x_1x_2} = -\frac{x_2}{2x_1} \qquad \textit{For calculations, we will use } MRS = \frac{MU_1}{MU_2}$$

6.C. Interpret the MRS economically.

- The MRS represents the consumer's marginal rate of substitution between the two goods.
- Specifically, it measures the maximum number of units of good 2 the consumer is willing to give up in exchange for one additional unit of good 1, holding utility constant.

6.D. As the consumer increases consumption of x_1, does the MRS increase, decrease, or remain constant? Explain briefly.

- As x_1 increases, the MRS decreases, since $MRS = \frac{x_2}{2x_1}$ is decreasing in x_1. This reflects diminishing marginal willingness to substitute good 2 for good 1.

Problem 7. Preferences and Indifference Curves

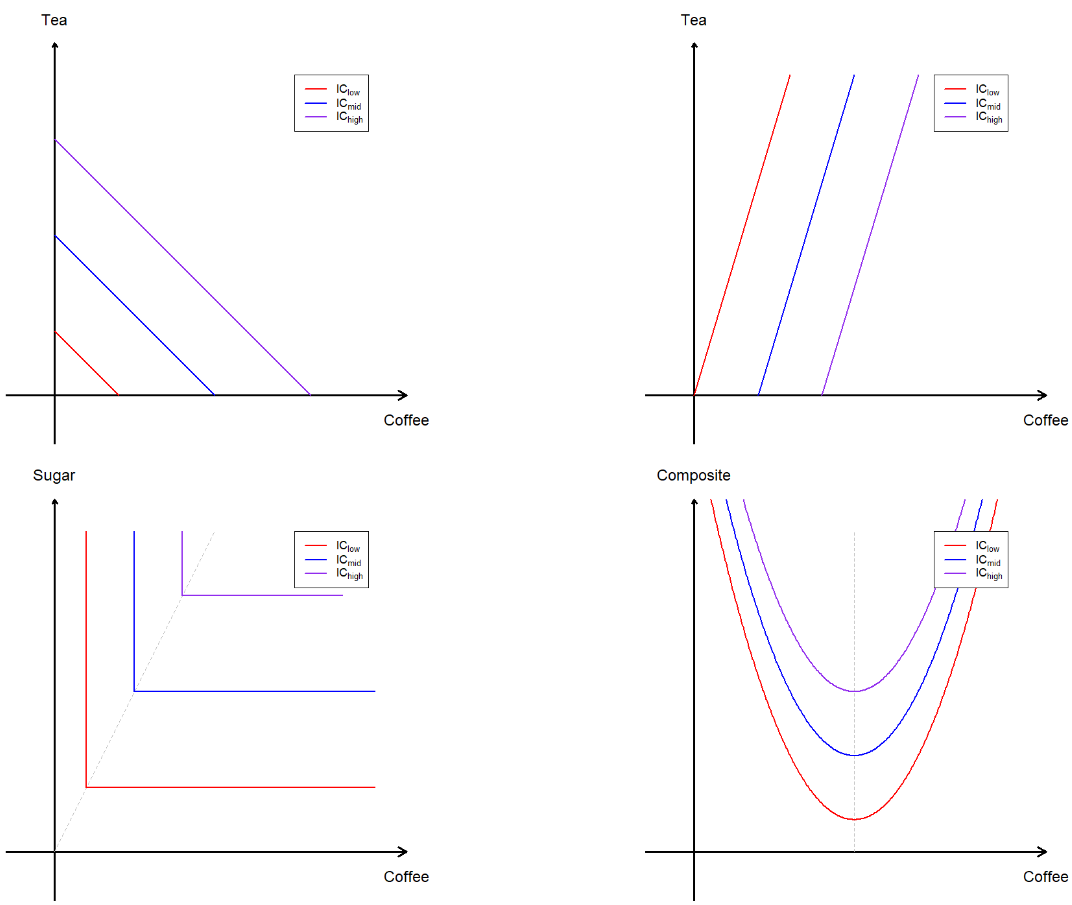
Plot two indifference curves each, resulting from the following preferences, and indicate which indifference curve represents bundles that the consumer prefers over the other:

7.A. The consumer cannot tell the difference between Coffee and Tea.

7.B. The consumer likes Coffee, but dislikes Tea.

7.C. The consumer always takes 2 teaspoons of Sugar with each cup of Coffee.

7.D. The consumer enjoys Coffee up to 5 cups a day, but past that it causes discomfort.



Problem 8. Optimal Choice

Suppose a consumer has well-behaved preferences and faces a linear budget constraint.

8.A. Explain what it means for a bundle to be utility-maximizing.

- A bundle (x_1^*, x_2^*) is utility-maximizing if it is feasible (it lies in the consumer's budget set) and it is at least as preferred as every other feasible bundle. Formally:
  - It satisfies the budget constraint: $p_1 x_1^* + p_2 x_2^* \leq m$
  - For any other feasible bundle (x_1, x_2), we have $(x_1^*, x_2^*) \succsim (x_1, x_2)$

8.B. Describe the tangency condition between the budget constraint and an indifference curve.

- When the utility-maximizing bundle is an interior solution (both goods are consumed in positive amounts) and preferences are smooth, the optimal bundle occurs at a point where the budget line is tangent to the highest attainable indifference curve.
- At the optimum, the consumer's willingness to trade good 2 for good 1 (the slope of the indifference curve in absolute value) equals the market's required trade-off (the slope of the budget line in absolute value). If these slopes were not equal, the consumer could reallocate spending to reach a higher indifference curve while remaining feasible.

8.C. Under what circumstances might a utility-maximizing bundle occur at a corner solution?

- A corner solution occurs when the optimal choice lies at an intercept of the budget line, meaning the consumer buys only one good (or zero of one good). This can happen when:
  - Preferences are such that the consumer strongly favors one good, for example with (near) perfect substitutes, where the consumer wants to spend all income on the good that gives higher "bang for the buck."
  - The tangency condition cannot hold, because the highest attainable indifference curve touches the budget set at an endpoint rather than at a tangency point.
  - Indifference curves have kinks or are not differentiable, such as perfect complements. In those cases, the optimum may occur at a kink (not a tangency) or at a boundary point depending on the budget line.

Problem 9. The Utility Maximization Problem: Cobb-Douglas Preferences

Suppose a consumer's utility function is given by $u(x_1, x_2) = 4x_1x_2^2$. The unit prices of good 1 and good 2 are given as $p_1 = 5$ and $p_2 = 4$, respectively. The consumer's income is $m = 60$, and the consumer allocates income across two goods only.

9.A. Using your answers from 6.A and 6.B, express the first-order condition for the consumer's utility maximization.

$$MRS = \frac{p_1}{p_2} \quad \Rightarrow \quad \frac{4x_2^2}{8x_1x_2} = \frac{5}{4} \quad \Rightarrow \quad \boxed{\frac{x_2}{2x_1} = \frac{5}{4}} \quad \Rightarrow \quad x_2 = \frac{5}{2}x_1$$

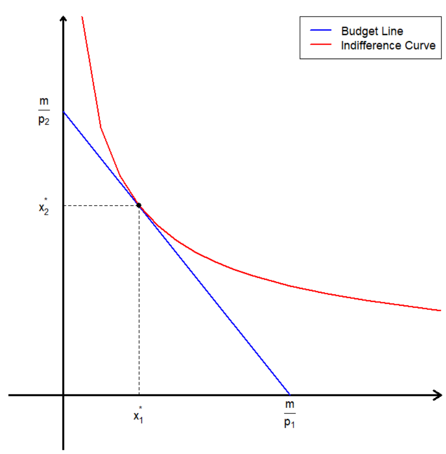
9.B. Solve the first-order condition from 9.A together with the budget constraint to find the utility-maximizing bundle (x_1^*, x_2^*).

$$\begin{aligned} 5x_1 + 4x_2 = 60 \quad &\Rightarrow \quad 5x_1 + 4\left(\frac{5}{2}x_1\right) = 60 \quad \because x_2 = \frac{5}{2}x_1 \\ &\Rightarrow \quad 15x_1 = 60 \\ &\Rightarrow \quad x_1^* = 4 \\ &\Rightarrow \quad x_2^* = 10 \quad \because x_2 = \frac{5}{2}x_1 \end{aligned}$$

9.C. Verify that the utility-maximizing bundle exhausts the consumer's budget. Why does the utility-maximizing bundle exhaust the consumer's budget?

$$5x_1^* + 4x_2^* = 5 \times 4 + 4 \times 10 = 60 = m$$

9.D. On a single graph, draw the consumer's budget constraint and an (approximate) indifference curve passing through the optimal bundle. Clearly indicate the point of tangency.



Problem 10. Utility Maximization: Perfect Substitutes

Suppose a consumer's utility function is given by $u(x_1, x_2) = 3x_1 + 2x_2$. The unit prices of good 1 and good 2 are given as $p_1 = 2$ and $p_2 = 2$, respectively. The consumer's income is $m = 50$, and the consumer allocates income across two goods only.

10.A. Find the consumer's marginal utility with respect to goods 1 and 2, respectively.

$$MU_1 = \frac{\partial u}{\partial x_1} = 3, \quad MU_2 = \frac{\partial u}{\partial x_2} = 2$$

10.B. Find the consumer's marginal rate of substitution.

$$MRS \equiv \frac{MU_1}{MU_2} = \frac{3}{2} \qquad \textit{Note: Technically, } MRS \equiv -\frac{MU_1}{MU_2}$$

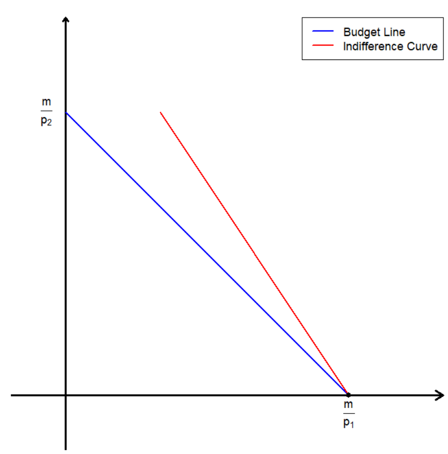
10.C. Find the equation expressing the consumer's budget line.

$$p_1x_1 + p_2x_2 = m \quad \Rightarrow \quad \boxed{2x_1 + 2x_2 = 50}$$

10.D. State the necessary condition characterizing the consumer's utility-maximizing bundle for these preferences.

- If $\frac{MU_1}{p_1} > \frac{MU_2}{p_2}$, spend all income on good 1 (a corner solution with $x_2 = 0$).
- If $\frac{MU_1}{p_1} < \frac{MU_2}{p_2}$, spend all income on good 2 (a corner solution with $x_1 = 0$).
- If $\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$, any bundle on the budget line is utility-maximizing.

10.E. Solve the necessary condition from `10.D.` together with the budget constraint from `10.C.` to find the utility-maximizing bundle (x_1^*, x_2^*) and draw the consumer's budget constraint and the indifference curve passing through the optimal bundle.



Problem 11. Utility Maximization: Perfect Complements

Suppose a consumer's utility function is given by $u(x_1, x_2) = min\{x_1, 2x_2\}$. The unit prices of good 1 and good 2 are given as $p_1 = 5$ and $p_2 = 2$, respectively. The consumer's income is $m = 50$, and the consumer allocates income across two goods only.

11.A. Describe the shape of the consumer's indifference curves.

- This represents perfect complements. Indifference curves are L-shaped (right angles). The kink occurs where the two arguments inside the minimum are equal: $x_1 = 2x_2$.

11.B. Is the marginal rate of substitution well-defined for these preferences? Explain briefly.

- MRS is not well-defined at the kink, and tangency conditions are generally not the right tool.

11.C. Find the equation expressing the consumer's budget line.

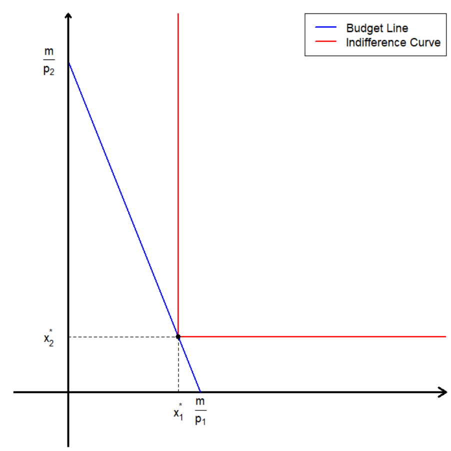
$$p_1x_1 + p_2x_2 = m \quad \Rightarrow \quad \boxed{5x_1 + 2x_2 = 50}$$

11.D. State the necessary condition characterizing the consumer's utility-maximizing bundle for these preferences.

$$x_1 = 2x_2$$

11.E. Solve the necessary condition from `11.D.` together with the budget constraint from `11.C.` to find the utility-maximizing bundle (x_1^*, x_2^*) and draw the consumer's budget constraint and the indifference curve passing through the optimal bundle.

$$
\begin{aligned}
& 5x_1 + 2x_2 = 50 \\
\Rightarrow\ & 5(2x_2) + 2x_2 = 50 \quad \because x_1 = 2x_2 \\
\Rightarrow\ & 12x_2 = 50 \\
\Rightarrow\ & x_2^* = \frac{25}{6} \\
\Rightarrow\ & x_1^* = \frac{25}{3} \quad \because x_1 = 2x_2
\end{aligned}
$$



Problem 12. Feedback Questions

These questions are graded only on completion, and your responses to this question will be used to improve pacing and explanations in upcoming lectures.

12.A. Which part(s) of this problem set did you find challenging? (Select all that apply.)

- Budget constraints and comparative statics
- Preferences and indifference curves
- Utility functions and MRS
- Solving the utility maximization problem
- Interpreting graphs and optimal bundles

Briefly explain why you found this part challenging (1–2 sentences).

12.B. At this point in the course, which statement best describes you?

- I understand the ideas conceptually but struggle with the math
- I can do the math but do not always understand what it means
- I am comfortable with both the math and the intuition
- I feel lost and am not sure where my confusion starts

Briefly say what would help most in class.

- Score: ______________________
- Date: ______________________