CONCORDIA COLLEGE
MOORHEAD, MINNESOTA

- Name: ________________________
- Date: ________________________

BUSN 301: Intermediate Microeconomic Theory

Practice Final Exam: Suggested Solutions

Spring 2026

INSTRUCTIONS:

- This practice final exam is designed to help you review your understanding of the course material.
- The practice final exam is not graded, and its results do not affect your final course grade.

Problem 1. Consumer Theory

Suppose that a consumer has preferences over two goods x_1 and x_2, represented by the utility function:

$$u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

The prices of the two goods are $p_1 = 1$ and $p_2 = 2$, respectively, and the consumer's income is $m = 20$.

1.A. Consider the bundle $(x_1, x_2) = (8, 6)$. Is this bundle affordable? Show your work.

- The bundle $(x_1, x_2) = (8, 6)$ is affordable.
- The budget constraint can be expressed as $p_1 x_1 + p_2 x_2 \leq m$.
- Substituting the terms, we find that $1 \cdot 8 + 2 \cdot 6 = 20 \leq 20$.

1.B. Compute the marginal rate of substitution at the bundle $(x_1, x_2) = (8, 6)$.

$$MU_1 \equiv \frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$MU_2 \equiv \frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}$$

$$MRS \equiv \frac{MU_1}{MU_2} = \frac{\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}}{\frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}} = \frac{x_2}{x_1}$$

$$\boxed{MRS = \frac{3}{4}}$$

1.C. Is the bundle $(x_1, x_2) = (8, 6)$ optimal for the consumer? Explain briefly using the first-order condition.

- The bundle $(x_1, x_2) = (8, 6)$ is not optimal.
- The relevant first-order condition for an interior optimum is $MRS = \frac{p_1}{p_2}$.
- From `1.B`, we have $MRS = \frac{x_2}{x_1}$.
- Evaluated at $(8, 6)$, this gives $MRS = \frac{6}{8} = \frac{3}{4} \neq \frac{1}{2} = \frac{p_1}{p_2}$.

Problem 1. Consumer Theory (continued)

1.D. If the bundle is not optimal, describe how the consumer would adjust consumption. Should they consume more of good 1 or good 2?

- The marginal rate of substitution (roughly) measures how many units of x_2 the consumer is willing to give up for 1 extra unit of x_1.
- The price ratio indicates how many units of x_2 the consumer must give up for 1 extra unit of x_1 in the market.
- Since the consumer is willing to give up more units of x_2 than the market demands for an additional unit of x_1, the consumer should decrease their consumption of x_2 and increase their consumption of x_1.

1.E. Find the consumer's optimal bundle (x_1^*, x_2^*).

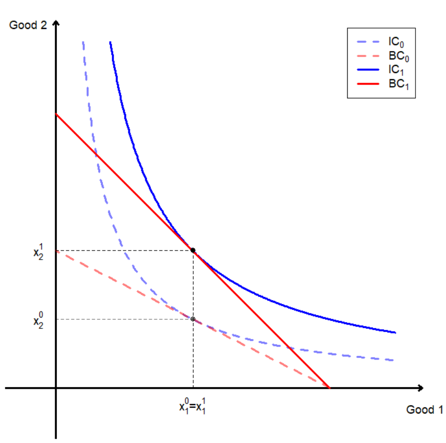
$$MRS = \frac{p_1}{p_2} \quad \Rightarrow \quad \frac{x_2^*}{x_1^*} = \frac{1}{2} \quad \Rightarrow \quad x_1^* = 2x_2^*$$

Plugging the optimal ratio into the budget constraint, we obtain the consumer's optimal bundle.

$$p_1 x_1^* + p_2 x_2^* = m \quad \Rightarrow \quad x_1^* + 2x_2^* = 20 \quad \Rightarrow \quad x_1^* + x_1^* = 20 \quad \Rightarrow \quad \boxed{x_1^* = 10} \quad \Rightarrow \quad \boxed{x_2^* = 5}$$

1.F. Suppose that the price of good 2 decreases from $p_2 = 2$ to $p_2 = 1$, holding other prices and income fixed.

- How does the optimal consumption of good 2 change?
- Illustrate this change in a graph with budget constraints and indifference curves. A rough but clearly labeled sketch is sufficient. Be sure to label both budget constraints, the optimal bundles, the corresponding indifference curves, and the axes.



- The consumer's optimal quantity of good 2 increases

Problem 2. Producer Theory

Suppose that a firm produces output q using two inputs x_1 and x_2, according to the production function:

$$f(x_1, x_2) = 3x_1 x_2^{\frac{1}{2}}$$

The prices of the two inputs are $w_1 = 2$ and $w_2 = 1$, respectively. Suppose that the firm wishes to produce $\bar{q} = 24$.

2.A. Compute the marginal product of each factor of production.

$$MP_1 \equiv \frac{\partial f(x_1, x_2)}{\partial x_1} = 3x_2^{\frac{1}{2}}$$
$$MP_2 \equiv \frac{\partial f(x_1, x_2)}{\partial x_2} = \frac{3}{2} x_1 x_2^{-\frac{1}{2}}$$

2.B. Consider the input bundle $(x_1, x_2) = (4, 4)$. Does this bundle produce $\bar{q} = 24$? Show your work.

- The bundle $(x_1, x_2) = (4, 4)$ can produce $\bar{q}$.
- Substituting the terms into the production function, $f(4, 4) = 3 \cdot 4 \cdot 4^{\frac{1}{2}} = 24$.

2.C. Compute the marginal rate of technical substitution of input 1 for input 2.

Taking the answers from `2.A`, we find:

$$MRTS_{1,2} = \frac{MP_1}{MP_2} = \frac{3x_2^{\frac{1}{2}}}{\frac{3}{2} x_1 x_2^{-\frac{1}{2}}} = \frac{2x_2}{x_1}$$

Problem 2. Producer Theory (continued)

2.D. Evaluate the MRTS at the bundle $(4,4)$. Is this bundle cost-minimizing? Explain briefly using the first-order condition.

Taking the answers from `2.C`, and evaluating at bundle $(4,4)$, we find that the bundle satisfies the first-order condition, and therefore is cost-minimizing:

$$MRTS_{1,2} = \frac{2 \cdot 4}{4} = 2 = \frac{2}{1} = \frac{w_1}{w_2}$$

2.E. If the bundle is not cost-minimizing, describe how the firm should adjust its input use.

- From `2.D`, we find that $MRTS = \frac{w_1}{w_2}$, so the bundle $(4,4)$ is cost minimizing.
- Hypothetical scenario if $MRTS < \frac{w_1}{w_2}$: Since the technology allows the producer to give up less x_2 than the market requires for an additional unit of x_1, the producer should decrease the use of x_1 and increase the use of x_2.
  - MRTS: How many units of x_2 can be substituted for one additional unit of x_1 while maintaining the same level of production, as determined by production technology.
  - Price ratio: How many units of x_2 the producer must give up for one additional unit of x_1, as determined by the factor market.

2.F. Find the firm's cost-minimizing input bundle (x_1^*, x_2^*)

$$MRTS = \frac{w_1}{w_2} \quad \Rightarrow \quad \frac{2x_2^*}{x_1^*} = 2 \quad \Rightarrow \quad x_2^* = x_1^*$$

Plugging the optimal ratio into the production quota, we find the producer's optimal input bundle.

$$3x_1^*(x_2^*)^{\frac{1}{2}} = 24 \quad \Rightarrow \quad 3x_1^*(x_1^*)^{\frac{1}{2}} = 24 \quad \Rightarrow \quad \boxed{x_1^* = 4} \quad \Rightarrow \quad \boxed{x_2^* = 4}$$

2.G. Suppose that the price of input 1 increases, holding the price of input 2 fixed. How would the firm's optimal input mix change? Explain briefly.

- If the price of input 1 rises, the factor price ratio $\frac{w_1}{w_2}$ increaes.
- The first-order condition requires that $MRTS = \frac{w_1}{w_2}$, so the optimal ratio $\frac{x_2}{x_1}$ also increases.
- The firm substitutes away from input 1 and toward input 2.

Problem 3. Cost and Firm Behavior

Suppose that a firm has the following total cost function:

$$c(y) = y^2 + 2y + 9$$

3.A. Compute the firm's marginal cost (MC), average variable cost (AVC), and average total cost (ATC) functions.

$$MC \equiv \frac{\partial c(y)}{\partial y} = 2y + 2$$
$$AVC \equiv \frac{c_v(y)}{y} = \frac{y^2 + 2y}{y} = y + 2$$
$$ATC \equiv \frac{c(y)}{y} = \frac{y^2 + 2y + 9}{y} = y + 2 + \frac{9}{y}$$

3.B. At what level of output is the firm's average total cost (ATC) minimized?

Find the value of y that sets the derivative of the ATC to zero:

$$\frac{dATC}{dy} = 0 \quad \Rightarrow \quad 1 - \frac{9}{y^2} = 0 \quad \Rightarrow \quad \boxed{y = 3}$$

3.C. Suppose that the market price of the output is given as $p = 10$. Without computing profit immediately, determine whether the firm is earning positive profit, zero profit, or negative profit. Explain briefly.

- When producing at the ATC-minimizing output level $y = 3$, the firm's ATC is 8.
- Since $p > \min ATC$, the firm is making positive profits.

3.D. At $p = 10$, solve for the firm's profit-maximizing output level.

The first-order condition for profit maximization is:

$$p = MC \quad \Rightarrow \quad 10 = 2y^* + 2 \quad \Rightarrow \quad \boxed{y^* = 4}$$

Problem 3. Cost and Firm Behavior (continued)

3.E. At price $p = 10$, compute the firm's profit.

Using the answer from `3.D`, we set up the firm's profit function:

$$\pi = p \cdot y - c(y) \quad \Rightarrow \quad \pi = 10 \cdot 4 - (4^2 + 2 \cdot 4 + 9) \quad \Rightarrow \quad \boxed{\pi = 7}$$

3.F. Suppose that the market price decreased to $p = 3$. Should the firm continue producing or shut down in the short run? Explain briefly.

- The firm's shutdown price $\min AVC = 2$.
- Since $p > \min AVC$, the firm does not shut down in the short run.

3.G. Suppose firms in this market are free to enter and exit. If firms like this one are earning positive profit in the short run, what would you expect to happen over time in the long run? Briefly explain.

- When firms in the market earn positive profits, it attracts entrants that can replicate a similar production technology.
- Each new entrant shifts the market supply curve to the right, decreasing the market prices in the process.
- This decrease in market prices causes each individual firm's profits to fall.
- New entrants enter until no firm in the market earns positive profits, and profits fall to zero in the long run.

3.H. Comparing the short-run and long-run supply curves, what characteristics would you expect to observe?

- The long-run market supply curve is generally flatter than the short-run market supply curve.
- In extreme environments with identical technologies and perfect competition, the long-run market supply curve will be completely flat.

Problem 4. Market Equilibrium and Welfare

Suppose that the market demand and supply curves for a good are given by:

$$D(p) = 30 - p$$
$$S(p) = p - 6$$

4.A. Find the competitive equilibrium price and quantity.

The market is in equilibrium when the quantity demanded matches quantity supplied:

$$D(p^*) = S(p^*) \quad \Rightarrow \quad 30 - p^* = p^* - 6 \quad \Rightarrow \quad \boxed{p^* = 18} \quad \Rightarrow \quad \boxed{q^* = 12}$$

4.B. Suppose the government introduces a per-unit subsidy of $s = 4$. Write the equation that relates the price paid by consumers, the price received by sellers, and the subsidy s.

The consumers pay s less than what the producers receive:

$$p_d = p_s - s \quad \Rightarrow \quad \boxed{p_d = p_s - 4}$$

4.C. Find the new equilibrium quantity, the price paid by consumers, and the price received by sellers after the subsidy.

The market is in equilibrium when the quantity demanded matches quantity supplied:

$$D(p_d) = S(p_s) \quad \Rightarrow \quad 30 - p_d = p_s - 6$$

Substitute either p_d or p_s with the relationship found in 4.B:

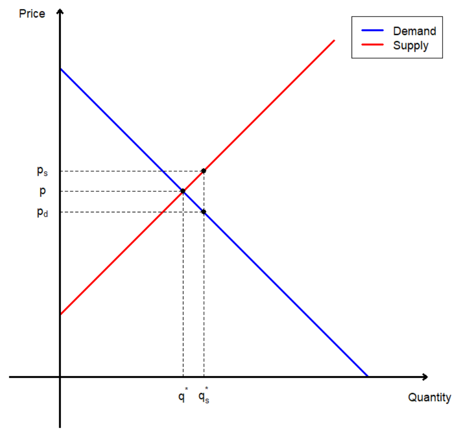
$$30 - p_d = p_s - 6 \quad \Rightarrow \quad 30 - (p_s - 4) = p_s - 6 \quad \Rightarrow \quad \boxed{p_s = 20} \quad \Rightarrow \quad \boxed{p_d = 16}$$

Plug in the prices found above to the demand or supply function to find the post-subsidy quantity:

$$D(p_d) = 30 - p_d \quad \Rightarrow \quad \boxed{q_s^* = 14}$$

Problem 4. Market Equilibrium and Welfare (continued)

4.D. Illustrate the market before and after the subsidy in a graph. A rough but clearly labeled sketch is sufficient. Label the original equilibrium, the new equilibrium, the price paid by consumers, the price received by sellers, and the subsidy wedge.



4.E. Compute the deadweight loss created by the subsidy, as well as consumer surplus, producer surplus, and government expenditure.

Consumer and producer surplus before subsidies can be found as follows:

$$CS_0 = \frac{1}{2} \cdot (30 - 18) \cdot 12 = 72, \qquad PS_0 = \frac{1}{2} \cdot (18 - 6) \cdot 12 = 72$$

Consumer and producer surplus after subsidies can be found as follows:

$$CS_1 = \frac{1}{2} \cdot (30 - 16) \cdot 14 = 98, \qquad PS_1 = \frac{1}{2} \cdot (20 - 6) \cdot 14 = 98$$

The government's subsidy expenditure and deadweight loss can be found as follows:

$$\text{Expenditure} = 4 \cdot 14 = 56, \qquad DWL = \frac{1}{2} \cdot 4 \cdot 2 = 4$$

Problem 5. General Equilibrium

Suppose that there are two agents A and B, and two goods, 1 and 2, where the total endowment is fixed at $(\omega^1, \omega^2) = (100, 100)$. Each agent's utility functions are given as:

$$u_A(x_A^1, x_A^2) = (x_A^1)^\alpha (x_A^2)^{1-\alpha}, \qquad 0 < \alpha < 1$$
$$u_B(x_B^1, x_B^2) = (x_B^1)^\beta (x_B^2)^{1-\beta}, \qquad 0 < \beta < 1$$

The economy's initial endowment is given as $(\omega_A^1, \omega_A^2, \omega_B^1, \omega_B^2) = (50, 50, 50, 50)$.

5.A. Write down the marginal rate of substitution (MRS) for each consumer.

Here, we find agent A's MRS. Agent B's MRS can be found following the same steps:

$$MU_A^1 \equiv \frac{\partial u_A(x_A^1, x_A^2)}{\partial x_A^1} = \alpha \cdot (x_A^1)^{\alpha-1}(x_A^2)^{1-\alpha}$$
$$MU_A^2 \equiv \frac{\partial u_A(x_A^1, x_A^2)}{\partial x_A^2} = (1-\alpha) \cdot (x_A^1)^{\alpha}(x_A^2)^{-\alpha}$$
$$MRS_A \equiv \frac{MU_A^1}{MU_A^2} = \frac{\alpha(x_A^1)^{\alpha-1}(x_A^2)^{1-\alpha}}{(1-\alpha)\cdot(x_A^1)^{\alpha}(x_A^2)^{-\alpha}} = \frac{\alpha}{1-\alpha} \cdot \frac{x_A^2}{x_A^1}$$

5.B. Suppose that $\alpha \neq \beta$. Starting from the initial endowment, would you expect gains from trade to exist? Explain briefly using marginal rates of substitution.

For an interior Pareto efficient allocation in this exchange economy, we require $MRS_A = MRS_B$, and measured at $(50, 50, 50, 50)$:

$$MRS_A = \frac{\alpha}{1-\alpha} \neq \frac{\beta}{1-\beta} = MRS_B$$

So the allocation is not Pareto optimal, and we should expect there to exist gains from trade.

5.C. Explain how differences in preferences (i.e., $\alpha \neq \beta$) affect the direction of trade between the two consumers.

- If $\alpha > \beta$, then A has a relatively stronger preference for good 1, so A should be willing to give up more of good 2 in exchange for good 1.
  - Agent A tends to buy/import good 1.
  - Agent A tends to sell/export good 2.
  - Agent B does the opposite.
- If $\alpha < \beta$, then B values good 1 relatively more than A does. So B will demand more of good 1 and offer good 2 in trade, while A will do the opposite.

Problem 5. General Equilibrium (continued)

5.D. Suppose the relative price of good 1 is $p_1/p_2 = p$. Describe how each consumer's demand for good 1 depends on p and their preference parameter (α or β).

With Cobb-Douglas preferences, each consumer spends a constant share of income on good 1:

$$x_A^1 = \alpha \cdot \frac{m_A}{p_1}, \qquad x_B^1 = \beta \cdot \frac{m_B}{p_1}$$

- Demand for good 1 is decreasing in its relative price $p = \frac{p_1}{p_2}$.
- Demand for good 1 increases with α (for A) and β (for B).

5.E. In equilibrium, what condition must hold between the two consumers' marginal rates of substitution and the price ratio? Explain briefly.

$$MRS_A = MRS_B = \frac{p_1}{p_2}$$

This holds because each consumer sets their marginal rate of substitution equal to the market price ratio.

5.F. Explain how prices adjust if there is excess demand for good 1.

- If there is excess demand for good 1, then good 1 is relatively scarce at the current price.
- This creates upward pressure on the price of good 1, and p_1 rises relative to p_2, meaning the relative price $\frac{p_1}{p_2}$ increases.
- As good 1 becomes more expensive, consumers demand less of good 1, and the market moves toward equilibrium.

5.G. Would you expect the equilibrium allocation to depend on the initial distribution of endowments? Briefly explain your reasoning.

Yes. In a pure exchange economy, the initial distribution of endowments affects each consumer's income, since income is equal to the market value of their endowment.

$$m_A = p_1\omega_A^1 + p_2\omega_A^2, \qquad m_B = p_1\omega_B^1 + p_2\omega_B^2$$

Because demand depends on income, changing the initial endowment distribution generally changes the competitive equilibrium allocation. However, the equilibrium allocation will still be Pareto optimal, even though the particular point on the contract curve may differ depending on the initial endowment.